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A SPECTROPHOTOMETRIC STUDY OF THE LUMINOUS RADIATION FROM THE NERNST LAMP GLOWER UNDER VARYING CURRENT DENSITY.

By Leon W. Hartman.

THE character of the luminous radiation from different sources of light has occupied the attention of physicists for many years. The numerous problems related to this subject which have been studied represent widely diversified interests and extend over a wide range of investigation. Thus, in radiation, for example, the luminous radiation and the radiant efficiency of the magnesium light were studied by Rogers;[1] the luminous radiation from the Auer mantle was studied by Miss Hill;[2] that from zinc oxide by Nichols and Snow;[3] that from the lime light by Pickering,[4] by Nichols and Franklin,[5] and by Nichols and Miss Crehore;[6] the total radiation and the radiant efficiency of the mercury arc were studied by Arons,[7] Geer,[8] and Geer and Coblentz;[9] that from platinum has been studied by numerous investigators, among whom mention may be made of

[1] Rogers, Am. Jour. Sci., **43**, p. 301, 1892.
[2] Nichols, Lab. Manual, **2**, p. 373, 1894.
[3] Nichols and Snow, Phil. Mag., [5], **32**, p. 401, 1891.
[4] Pickering, Proc. Am. Acad. Arts and Sci., **15**, p. 236, 1880.
[5] Nichols and Franklin, Am. Jour. Sci., **38**, p. 100, 1889.
[6] Nichols and Miss Crehore, Phys. Rev., **2**, p. 161, 1894.
[7] Arons, Wied. Ann., **47**, p. 767, 1892; **58**, p. 73, 1896.
[8] Geer, Phys. Rev., **16**, p. 94, 1903.
[9] Geer and Coblentz, Phys. Zeitschr., **4**, p. 257, 1903.

Magnus,[1] Violle,[2] Draper,[3] Nichols,[4] Chatelier,[5] Becquerel,[6] Crova [7] and Zöllner.[8]

The radiation from heated pigments was studied by Nichols and Snow.[9] The radiation from different forms of carbon has been thoroughly and carefully studied by Weber,[10] Schumann,[11] Nichols and Franklin,[12] Nichols,[13] Blaker,[14] Stewart,[15] Janet,[16] Le Chatelier [17] and others. In perusing some of the work mentioned above, it occurred to the writer that a study of the radiation from the rare-earth oxides used in the "glower" of the Nernst lamp would be of theoretical interest and of practical importance. In a paper on the behavior of these "glowers" when carrying an electric current,[18] mention is made of a spectral and bolometric study of the radiation from these luminous sources, but apparently no work was done in this direction. Inasmuch as no published work has appeared on this phase of the radiation from the Nernst lamp glower, it was decided to investigate this problem.

Inasmuch as no spectrophotometer was available for this work, a Kirchhoff spectrometer having a collimator with two unilateral micrometer slits was adapted for the purpose. This instrument has been described elsewhere.[19] This spectrometer has a broad circular horizontal plate of metal mounted upon a massive hollow cast-iron

[1] Magnus, Ann. der Phys., **124**, p. 476, 1865.

[2] Violle, C. R., **88**, p. 171, 1879; **92**, pp. 866, 1204, 1881; **95**, p. 163, 1887; **94**, p. 734, 1892.

[3] Draper, Phil. Mag., [3], **30**, p. 345, 1847.

[4] Nichols, Am. Jour. Sci., **18**, p. 446, 1879.

[5] Chatelier, C. R., **114**, p. 470, 1892; Bul. Soc. Chim. Paris, **47**, p. 42, 1887.

[6] Becquerel, Annals de Chem. et de Physique, [3], **68**, p. 47, 1863; C. R., **55**, p. 876, 1863.

[7] Crova, C. R., **57**, p. 497, 1878.

[8] Pogg. Ann., **100**, p. 381, 1857; **109**, p. 256, 1869.

[9] Nichols and Snow, Phil. Mag.,[5], **32**, p. 401, 1891.

[10] Weber, Wied. Ann., **32**, p. 256, 1897; PHYS. REV., **2**, pp. 112, 197, 1894.

[11] Schumann, Electrotech. Zeitschr., **5**, p. 220, 1884.

[12] Nichols and Franklin, Am. Jour. Sci., **38**, p. 100, 1889.

[13] Nichols, Proc. Am. Acad. Arts and Sci., **37**, p. 73, 1901.

[14] Blaker, PHYS. REV., **13**, p. 345, 1901.

[15] Stewart, PHYS. REV., **13**, p. 257, 1901; **15**, p. 306, 1902.

[16] Janet, C. R., **123**, p. 690, 1896; **126**, p. 734, 1898.

[17] Le Chatelier, Jour. de Phys., [3], **1**, p. 203, 1892.

[18] Nernst and Wild, Zeitschr. für Electrochemie, **7**, p. 373, 1900.

[19] Wüllner, Lehrbuch der Experimentalphysik, Bd. 4, p. 168, 1899.

column, which in turn is supported by three massive feet. Mounted upon this plate is the collimator and the swinging arm which supports the observing telescope. This arm is rigidly attached to a revolving pivot passing downward through the vertical axis of the instrument. In a vertical plane beneath the plate is mounted a plane mirror which is rigidly attached to the pivot of the telescope arm; thus when the telescope is rotated about its axis, the mirror rotates about the same axis. Through an opening in the cast-iron support of the spectrometer plate a beam of light from an illumi-

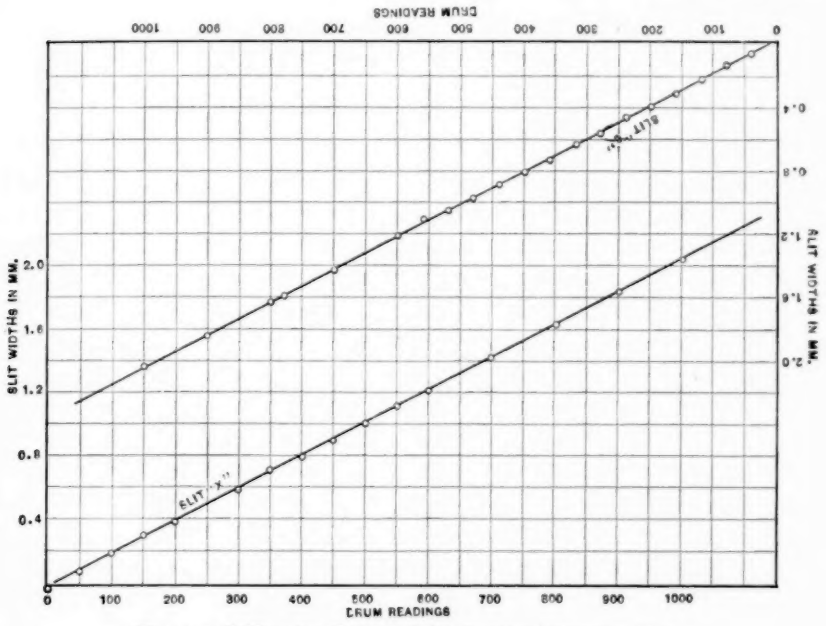

Fig. 1.—Calibration Curves of the Spectrophotometer Slits.

nated scale falls upon the mirror and is reflected into another observing telescope which is rigidly attached by metal supports to the lower surface of the circular plate as shown in Fig. 4.

The train of glass prisms belonging to the instrument was removed and replaced by a Rowland plane grating, having 14,437 lines to the inch, ruled on speculum metal. This grating was mounted in the axis of the instrument on a metal base having three leveling screws, and was therefore in the axis of rotation of the observing telescope. Thus an increased intensity in the less visible portions

of the spectrum was secured, and a normal spectrum and a straight line calibration curve were obtained. The collimator slits were then removed and calibrated under a high power microscope mounted on a Hilger dividing engine. Proper precautions were taken to avoid any error in measurement due to the "back lash" of the screw of the dividing engine or of the screws of the slits. The calibration curves given in Fig. 1 show that the screws of the slits are remarkably uniform and perfect. In Fig. 1 slit S refers to the slit which throughout was used with standard No. 1; slit X, therefore, was the slit before which was placed the Hefner lamp, the Nernst lamp and standard No. 2. In front of the upper slit of the collimator a totally reflecting prism was then mounted upon a blackened brass arm which swung about a vertical axis. By means of a screw adjustment this prism could be raised or lowered at will. The position of the prism in front of the collimator can be seen in Fig. 4. After remounting the slits the collimator and telescope were adjusted for parallel light. The collimator was then turned so that its axis passed normally through the vertical axis of the spectrometer, after which it was securely clamped in position. Thus, a beam of light passing through the collimator fell upon the middle portion of the Rowland grating. The observing telescope of the spectrometer, the ocular of which contained the cross-hairs and an adjustable slit so that only a narrow band of the spectrum could be observed at once, was then rigidly attached to its arm so that it had motion on this arm only in a vertical plane. As was noted above, this arm was attached to the axis of the instrument about which it could be swung in a horizontal plane. Thus the center of the grating was in the axis of rotation of the telescope. The spectrometer was then mounted in a dark room where this investigation was conducted. After the grating was adjusted so as to give a suitable intensity and dispersion in the spectrum of the first order, a scale divided into millimeters was suspended in front of the spectrometer mirror, by means of cords from one corner of the dark room. The spectrometer was then calibrated, by reference to the lines produced by vaporizing various metal salts in a Bunsen flame placed in front of the collimator slits. The calibration curve thus obtained was a straight line. The dispersion

was found to be sufficient to separate appreciably the two sodium lines and a difference of 0.001 μ was found to be equivalent to about 3.6 mm. on the scale. By virtue of certain changes that were made before any observations were taken several calibrations of the spectrometer were necessary, the final one of which is given in Fig. 2. The only difference found in these curves was a definite vertical shift throughout the whole length of the curve. Hence the calibration of the instrument could be tested at any time by reference to a sodium flame. As a precaution this was frequently done. As a

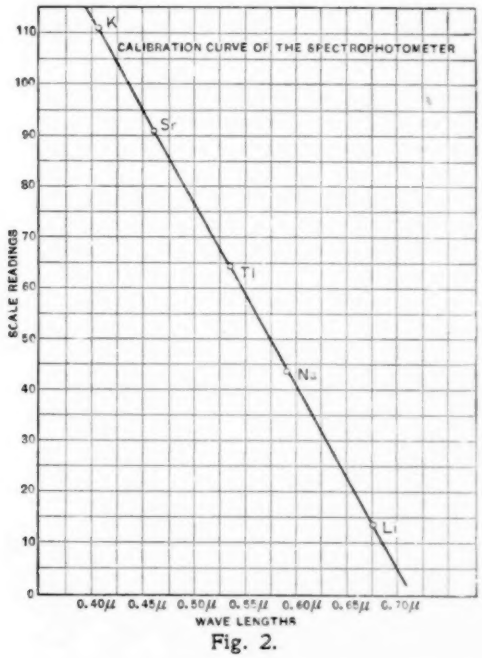

Fig. 2.

matter of fact the dark room in which this work was carried out was kept locked and no one but the writer was supposed to enter it, so that once adjustments were finally made, they were not disturbed by any one outside. The spectrometer observations and scale readings therefore never showed an appreciable difference from the calibration curve readings.

As a standard source of light an acetylene flame burning under constant pressure was used. The acetylene was produced in a Colt Automatic Acetylene Generator, in which finely divided calcium

carbide was automatically dropped into water. The gas thus generated was passed over into a gasometer whence it was conducted through a pressure regulator of the Moler pattern[1] into the dark room. By means of a glass **Y** placed in the line of gas tubing leading from the regulator, one branch of which lead to a **U**-tube water manometer, while the other branch lead to the burner, the gas pressure at the burner could be observed at will through a telescope placed near the observer at the spectrometer. To prevent evaporation from the manometer, the open end of the **U**-tube was closed with a cork having a small groove along the side. The acetylene was burned in a No. 4 Naphey burner[2] of the type shown in Fig. 3. In front of the flat flame of this burner and parallel to its plane was mounted a sheet of blackened tin containing a circular diaphragm 7.94 mm.[3] in diameter so that the opening was opposite the most uniform portion of the flame. This burner with diaphragm was then mounted to the right of the collimator slits with the sheet of tin facing the totally reflecting prism and its plane 10 cm. from the face of the prism. When this diaphragmed source of light was placed in the proper position before the reflecting prism, the burner was rigidly clamped in place and remained undisturbed throughout the whole investigation. It has been shown that such a source of light under constant pressure gives an intensity which remains uniform[4] and is therefore suitable for this investigation.

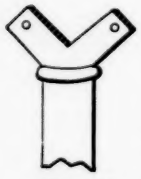
Fig. 3.

The Nernst lamp used was a 104-volt A. C. lamp. The "holder"[5] of the lamp was removed and the lamp was mounted on a support at a proper height so that the glower was horizontal and 100 cm. in front of the uncovered collimator slit. In series with the lamp was a rheostat (R) and a new accurately calibrated Weston A. C. ammeter (A), and to the platinum terminal wires of the glower were soldered the terminal wires of a recently calibrated Weston D. C. and A. C. voltmeter (V). Thus the voltages given in the data are

[1] Nichols, Jour. Frank. Inst., **150**, p. 359, 1900.

[2] A No. 4 Naphey burner is rated to burn one cubic foot of gas per hour.

[3] $\frac{5}{16}$ inches = 7.94 mm.

[4] Nichols, Jour. Frank. Inst., **150**, pp. 360, 386, 1900.

[5] The Nernst lamp and its parts have been fully described by Mr. A. J. Wurts, Trans. Am. Inst. Elect. Eng., **18**, p. 545, 1901.

those between the ends of the glower under varying current. It may be noted in passing that the lamp in this horizontal position was not entirely automatic, but inasmuch as the holder of the lamp had been removed, the heating circuit in each case was closed by hand, and when the glower became conducting the heating circuit was opened automatically. The arrangement of the apparatus is shown in Fig. 4.

In order to secure an alternating current of constant potential, a direct current of 110 volts was connected to the terminals of a 110-volt storage battery, and from the leads of the battery a line was run to the terminals of a motor belted to a line of shafting, from which was furnished the power to run the alternator and its exciter. Thus the A. C. voltage could be kept constant at any desired value

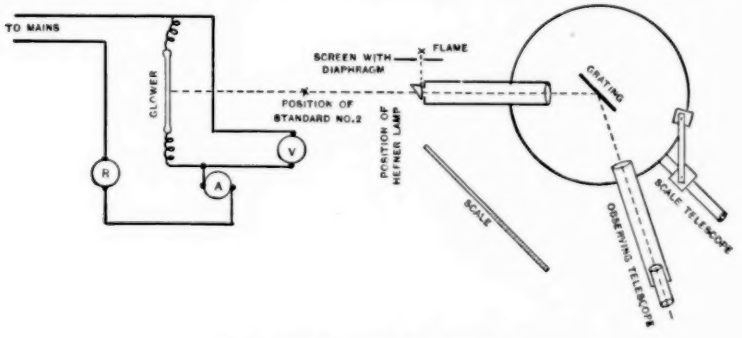

Fig. 4.—Diagram of Apparatus.

below its maximum. When the maximum value was reached, current was taken from the city A. C. mains which entered the laboratory.

To obtain results in terms of definite known units the Hefner-Altneck standard lamp[1] was used. That is, the diaphragmed acetylene flame used as a standard was determined in terms of the Hefner standard. This had previously been done, and the results obtained here are concordant with the earlier measurements. As has been pointed out it is desirable to compare the acetylene flame with the Hefner standard since in the latter the flame is produced by the combustion of a fuel of definite chemical composition under very nearly constant conditions.[2]

[1] Zeitschr. für Instrumentenkunde, 13, pp. 257–265, 1893.

[2] Nichols, Jour. Frank. Inst., 150, p. 371, 1900.

The method of taking the observations was as follows: Ten centimeters in front of the uncovered collimator slit, the Hefner lamp was adjusted in position. The lamp was allowed to burn twenty minutes or half an hour and then adjusted to the proper flame height, after which readings at various points in the spectrum were taken. Throughout the whole work eight or ten, and sometimes thirty or forty, readings were taken at each setting of the telescope. The mean of these readings was then taken for the position in question. Thus on different days a number of such curves were taken with the Hefner lamp, and the curves thus de-

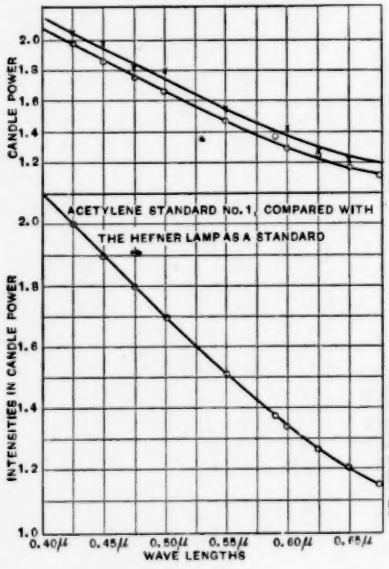

Fig. 5.

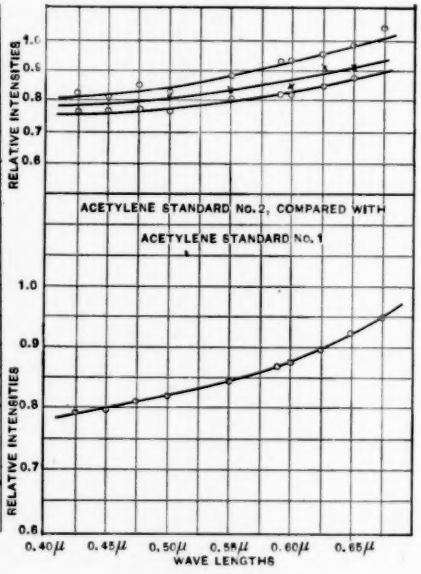

Fig. 6.

termined were found to agree so closely that a mean curve of the two agreeing most closely was taken. The latter is shown in Fig. 5. The data for these curves is given in Table I.

Fifty centimeters in front of the collimator slits was temporarily mounted another No. 4 Naphey burner having a blackened screen mounted in front of it with a circular diaphragm of exactly the same size as that used with the burner previously mentioned. This acetylene source is called standard No. 2; the former acetylene source, standard No. 1. The spectrophotometric curve of standard No. 2 in terms of No. 1 was then determined. This curve, shown

TABLE I.

Comparison of Standard No. 1 with Hefner Lamp.

Wave-Lengths.	Barometric Pressure = 764.4 mm. P = 7.4 cm. Psychrometer Readings { Dry Bulb = 23.2° C. Wet Bulb = 15.5° C.						Barometric Pressure = 764.9 mm, P = 7.4 cm. Psychrometer Readings { Dry Bulb = 22.5° C. Wet Bulb = 15.8° C.						
	Drum Readings of Micrometer.		Slit Widths in mm.		Ratio of Intensities in Hefners.	Ratio of Intensities in C. P.	Drum Readings of Micrometer.		Slit Widths in mm.		Ratio of Intensities in Hefners.	Ratio of Intensities in C. P.	Mean Ratio of Intensities in Candle Power from Curves.
	S_I	S_{II}	S_I	S_{II}			S_I	S_{II}	S_I	S_{II}			
λ=0.425 μ	73.44	150	0.121	0.287	2.37	2.05	75.25	150	0.125	0.287	2.30	1.98	2.010
λ=0.450 μ	74.90	150	0.125	0.287	2.30	1.99	78.88	150	0.133	0.287	2.16	1.86	1.900
λ=0.475 μ	80.11	150	0.1365	0.287	2.10	1.82	82.40	150	0.141	0.287	2.04	1.752	1.800
λ=0.500 μ	81.11	150	0.1380	0.287	2.08	1.80	86.00	150	0.149	0.287	1.925	1.658	1.701
λ=0.550 μ	92.00	150	0.1610	0.287	1.78	1.54	94.75	150	0.167	0.287	1.72	1.480	1.515
λ=0.589 μ	99.87	150	0.179	0.287	1.605	1.39	102.13	150	0.179	0.287	1.603	1.378	1.372
λ=0.600 μ	98.30	150	0.176	0.287	1.640	1.42	104.9	150	0.190	0.287	1.51	1.30	1.340
λ=0.625 μ	107.40	150	0.195	0.287	1.470	1.272	107.4	150	0.195	0.287	1.470	1.262	1.268
λ=0.650 μ	109.70	150	0.200	0.287	1.435	1.242	114.4	150	0.210	0.287	1.368	1.175	1.205
λ=0.675 μ	126.25	150	0.236	0.287	1.217	1.052	118.8	150	0.220	0.287	1.304	1.125	1.147

in Fig. 6, is the mean of several sets of independent observations taken on different days. This curve was obtained for the purpose of eliminating the absorption of the totally reflecting prism, which was not only marked but was also selective, as will be seen by referring to Fig. 6. Obviously, therefore, when the Nernst lamp is compared with a bare acetylene flame the results will be different from those obtained in the comparison with standard No. 1 in which the use of the reflecting prism is necessary.

Lastly, the Nernst lamp glower while carrying currents of different densities was compared spectrophotometrically with standard No. 1. At each observation throughout the whole series the current strength and the gas pressure were observed to see that they remained constant while a given set of observations was being taken. From eight to ten readings were taken for each point on the curve. In some cases as many as fifty observations were made for each point and then the mean was taken. The gas pressure was most conveniently kept constant at 7.4 cm. of water pressure.

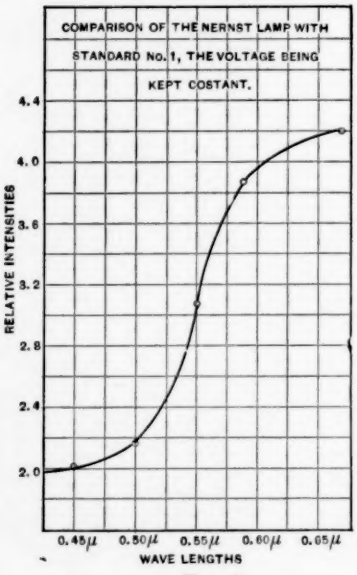

Fig. 7.

In using the Hefner-Altneck lamp corrections for humidity and barometric pressure according to Liebenthal's method[1] were made. Humidity was determined by observing the wet and dry bulb psychrometer and by using the values of vapor pressure obtained from Regnault's measurements.[2]

In taking the measurements on the Nernst lamp the first observations were made on another glower than on the one used throughout the series of measurements given in this paper. It was the original intention to obtain the spectrophotometric curves with the voltage maintained constant for a given set of observations. This, however, proved to be unsatisfactory. As an

[1] Liebenthal, Electroteck. Zeitschr., **16**, p. 655, 1895.

[2] Hempel's Gas. Analysis. Translated by Dennis, pp. 255, 373-4, 1892.

TABLE II.

Comparison of Standard No. 2 *with Standard No.* 1.

Wave-Lengths.	(1) $P=7.4$ cm. Drum Readings of Micrometer.		Slit Widths in mm.		Ratio of Intensities.	(2) $P=7.4$ cm. Drum Readings of Micrometer.		Slit Widths in mm.		Ratio of Intensities.	(3) $P=7.4$ cm. Drum Readings of Micrometer.		Slit Widths in mm.		Ratio of Intensities.	Mean Relative Intensity of 1, 2 and 3 from Curves.
	S_1	S_2	S_1	S_2		S_1	S_2	S_1	S_2		S_1	S_2	S_1	S_2		
$\lambda=0.425\,\mu$						89.30	100	0.152	0.183	0.830	82.6	100	0.141	0.183	0.770	0.793
$\lambda=0.450\,\mu$	86.6	100	0.150	0.183	0.820	86.33	100	0.150	0.183	0.820	82.8	100	0.1416	0.183	0.771	0.799
$\lambda=0.475\,\mu$	83.0	100	0.142	0.183	0.776	89.63	100	0.156	0.183	0.853	83.0	100	0.142	0.183	0.777	0.810
$\lambda=0.500\,\mu$	86.0	100	0.149	0.183	0.805	87.45	100	0.152	0.183	0.830	81.8	100	0.140	0.183	0.765	0.820
$\lambda=0.550\,\mu$	87.9	100	0.153	0.183	0.836	92.90	100	0.164	0.183	0.896	85.0	100	0.147	0.183	0.804	0.844
$\lambda=0.589\,\mu$						95.25	100	0.170	0.183	0.930	86.1	100	0.150	0.183	0.820	0.868
$\lambda=0.600\,\mu$	89.6	100	0.156	0.183	0.853	96.50	100	0.171	0.183	0.935	86.2	100	0.150	0.183	0.820	0.873
$\lambda=0.625\,\mu$	93.75	100	0.1655	0.183	0.905	97.75	100	0.174	0.183	0.950	89.3	100	0.1555	0.183	0.850	0.895
$\lambda=0.650\,\mu$	94.0	100	0.1660	0.183	0.907	100.55	100	0.180	0.183	0.985	91.1	100	0.160	0.183	0.875	0.923
$\lambda=0.675\,\mu$						105.75	100	0.1905	0.183	1.04_0						0.950

illustration, the curve shown in Fig. 7 was obtained in this manner. The readings began in the long waves and extended into the short ones. The potential difference was kept constant but no attention was paid to current strength. The curve obtained is a smooth curve but very different from those obtained later with constant current. On repeating these observations, passing through the spectrum in the reverse order, a curve of quite different shape was obtained. This method, therefore, was evidently unreliable. It is probable that the current strength gradually decreased in value with the time without giving an apparent change in voltage,[1] thus accidentally giving the curve mentioned. Thereafter, observations were taken with the current kept constant and a series of curves were obtained which were similar to those shown in Fig. 9.

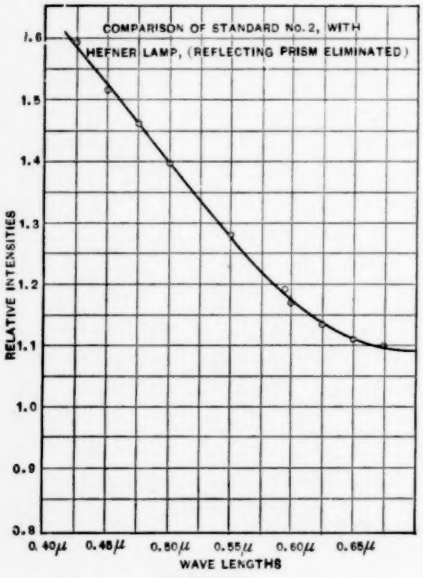

Fig. 8.

An accident, however, necessitated the remounting of a new glower. The first observations were taken at the rated normal current capacity of the lamp, namely, 0.8 amperes. Observations were then taken with decreasing currents down to 0.2 amperes. Then observations were taken with increasing currents; the maximum value reached was 1.50 amperes, at which point the platinum wire attached to the glower fused off after a few observations had been taken. The mean readings thus taken are recorded in Table III. in the order in which they were taken and are lettered accordingly, beginning *a*, *b*, *c*, . . . , etc. The curves obtained from these data are lettered likewise. No stress is laid upon the data obtained in the last set of readings and the curve obtained from these data is given only once — in Fig. 9 — after which it is not considered.

[1] This possibility is suggested by a characteristic curve of the glower given by Wurts, Trans. Am. Inst. Elect. Eng., **18**, p. 552, 1901.

TABLE III.

Comparison of Nernst Lamp with Standard No. 1.

Wave-Lengths.	(*a*) $V=86.2$; $I=0.80$; $P=7.4$ cm.					(*b*) $V=86.0$; $I=0.70$; $P=7.4$ cm.					(*c*) $V=84.0$; $I=0.60$; $P=7.4$ cm.				
	Drum Readings of Micrometer.		Slit Widths in mm.		Ratio of Intensities.	Drum Readings of Micrometer.		Slit Widths in mm.		Ratio of Intensities.	Drum Readings of Micrometer.		Slit Widths in mm.		Ratio of Intensities.
	S_1	S_n	S_1	S_n		S_1	S_n	S_1	S_n		S_1	S_n	S_1	S_n	
$\lambda=0.425\,\mu$						118.6	200	0.219	0.395	0.555	75.0	200	0.125	0.395	0.316
$\lambda=0.450\,\mu$	67.5	80	0.108	0.140	0.772	52.75	80	0.076	0.140	0.542	45.1	100	0.060	0.185	0.328
$\lambda=0.475\,\mu$						71.5	100	0.117	0.185	0.640	49.0	100	0.069	0.185	0.377
$\lambda=0.500\,\mu$	82.5	80	0.141	0.140	1.008	75.4	100	0.125	0.185	0.684	51.6	100	0.074	0.185	0.404
$\lambda=0.550\,\mu$	97.5	80	0.175	0.140	1.250	73.5	80	0.121	0.140	0.865	63.5	100	0.100	0.185	0.546
$\lambda=0.575\,\mu$											71.0	100	0.116	0.185	0.631
$\lambda=0.589\,\mu$	104.0	80	0.187	0.140	1.340	83.6	80	0.143	0.140	1.020	77.5	100	0.130	0.185	0.710
$\lambda=0.600\,\mu$											76.3	100	0.128	0.185	0.700
$\lambda=0.625\,\mu$	112.8	80	0.206	0.140	1.471	91.3	80	0.160	0.140	1.140					
$\lambda=0.650\,\mu$											84.5	100	0.145	0.185	0.793
$\lambda=0.675\,\mu$	122.9	80	0.228	0.140	1.630	126.0	100	0.235	0.183	1.281	92.1	100	0.162	0.185	0.885
	(*d*) $V=81.5$; $I=0.50$; $P=7.4$ cm.					(*e*) $V=77.9$; $I=0.40$; $P=7.4$ cm.					(*f*) $V=76.6$; $I=0.30$; $P=7.4$ cm.				
$\lambda=0.425\,\mu$	57.0	200	0.086	0.395	0.216	43.9	200	0.057	0.395	0.1430	44.0	400	0.057	0.800	0.0713
$\lambda=0.450\,\mu$	59.9	200	0.091	0.395	0.230	45.2	200	0.060	0.395	0.1580	41.6	400	0.053	0.800	0.0663
$\lambda=0.475\,\mu$	62.4	200	0.097	0.395	0.246	49.9	200	0.067	0.395	0.1696					
$\lambda=0.500\,\mu$	69.1	200	0.112	0.395	0.284	52.0	200	0.075	0.395	0.1900	49.1	400	0.069	0.800	0.0863
$\lambda=0.550\,\mu$	85.12 81.0	200	0.147 0.138	0.395	0.372 0.350	62.5	200	0.097	0.395	0.2456	53.6	400	0.078	0.800	0.0976
$\lambda=0.589\,\mu$	97.5	200	0.173	0.395	0.438	69.0	200	0.113	0.395	0.2860	61.5	400	0.095	0.800	0.1187
$\lambda=0.600\,\mu$	94.5	200	0.171	0.395	0.433	70.9	200	0.115	0.395	0.2910	65.2	400	0.103	0.800	0.1290
$\lambda=0.625\,\mu$															
$\lambda=0.650\,\mu$	108.6	200	0.198	0.395	0.501	80.6	200	0.136	0.395	0.3440	75.9	400	0.126	0.800	0.1575
$\lambda=0.675\,\mu$	116.3	200	0.215	0.395	0.545	86.4	200	0.150	0.395	0.3800	85.5	400	0.147	0.800	0.1837
$\lambda=0.700\,\mu$	133.0	200	0.250	0.395	0.633	98.6	200	0.175	0.395	0.4430	93.1	400	0.165	0.800	0.2062

Wave-Lengths.	(g) $V=82.0$; $I=0.20$ $P=7.4$ cm. Drum Readings of Micrometer.		Slit Widths in mm.		Ratio of Intensities.	(h) $V=78.5$; $I=0.40$; $P=7.4$ cm. Drum Readings of Micrometer.		Slit Widths in mm.		Ratio of Intensities.	(i) $V=87.0$; $I=0.80$; $P=7.4$ cm. Drum Readings of Microm.		Slit Widths in mm.		Ratio of Intensities.
	S_1	S_n	S_1	S_n		S_1	S_n	S_1	S_n		S_1	S_n	S_1	S_n	
$\lambda=0.425\,\mu$	41.1	1000	0.051	2.040	0.0250	68.4	400	0.110	0.800	0.1378	73.6	70	0.122	0.120	1.018
$\lambda=0.450\,\mu$	38.8	1000	0.046	2.040	0.0226	73.5	400	0.121	0.800	0.1515	80.2	70	0.136	0.120	1.133
$\lambda=0.500\,\mu$	40.7	1000	0.050	2.040	0.0245	76.6	400	0.128	0.800	0.1602	90.5	70	0.156	0.120	1.300
$\lambda=0.550\,\mu$	45.1	1000	0.060	2.040	0.0294	91.8	400	0.160	0.800	0.2000	113.5	80	0.207	0.141	1.470
$\lambda=0.589\,\mu$						111.2	400	0.203	0.800	0.2540	110.7	70	0.202	0.120	1.683
											127.0	80	0.240	0.141	1.701
$\lambda=0.600\,\mu$	47.9	1000	0.065	2.040	0.0319	113.0	400	0.207	0.800	0.2582	110.6	70	0.201	0.120	1.675
$\lambda=0.625\,\mu$											112.0	70	0.205	0.120	1.708
											110.8	70	0.203	0.120	1.692
$\lambda=0.650\,\mu$	57.7	1000	0.087	2.040	0.0426	131.8	400	0.248	0.800	0.3100	132.4	80	0.252	0.141	1.788
$\lambda=0.675\,\mu$	78.0	1000	0.131	2.040	0.0644	149.6	400	0.286	0.800	0.3580					
$\lambda=0.700\,\mu$						152.0	400	0.292	0.800	0.3650					

Wave-Lengths.	(j) $V=87.0$; $I=0.90$; $P=7.4$ cm. Drum Readings of Microm.		Slit Widths in mm.		Ratio of Intensities.	(k) $V=87.0$; $I=0.90$; $P=7.4$ cm. Drum Readings of Micrometer.		Slit Widths in mm.		Ratio of Intensities.
	S_1	S_2	S_1	S_2		S_1	S_2	S_1	S_2	
$\lambda=0.425\,\mu$	97.2	70	0.174	0.120	1.450	81.0	50	0.137	0.072	1.905
$\lambda=0.450\,\mu$	101.5	70	0.182	0.120	1.517	79.3	50	0.135	0.072	1.877
$\lambda=0.475\,\mu$						83.6	50	0.144	0.072	2.000
$\lambda=0.500\,\mu$	108.4	70	0.197	0.120	1.642	85.6	50	0.148	0.072	2.060
$\lambda=0.550\,\mu$	128.5	70	0.240	0.120	2.025					
$\lambda=0.589\,\mu$						94.8	50	0.172	0.072	2.385
$\lambda=0.600\,\mu$	134.5	70	0.253	0.120	2.108					
$\lambda=0.625\,\mu$	136.9	70	0.257	0.120	2.140	100.1	50	0.180	0.072	2.500
$\lambda=0.650\,\mu$	149.0	70	0.285	0.120	2.380	104.3	50	0.187	0.072	2.600
$\lambda=0.675\,\mu$	160.3	70	0.310	0.120	2.580	122.0	50	0.226	0.072	3.140
	(l) $V=86.8$; $I=1.00$; $P=7.4$ cm.					(m) $V=87.1$; $I=0.80$; $P=7.4$ cm.				
$\lambda=0.425\,\mu$	88.6	50	0.155	0.072	2.151	100	76.8	0.179	0.135	1.253
$\lambda=0.450\,\mu$	90.4	50	0.159	0.072	2.210					
$\lambda=0.475\,\mu$	91.8	50	0.161	0.072	2.239	100	71.9	0.179	0.124	1.441
$\lambda=0.500\,\mu$	97.3	50	0.173	0.072	2.400	100	69.9	0.179	0.129	1.390
$\lambda=0.550\,\mu$	104.5	50	0.189	0.072	2.622	100	63.9	0.179	0.108	1.656
$\lambda=0.589\,\mu$						100	60.8	0.179	0.101	1.772
$\lambda=0.600\,\mu$	108.4	50	0.197	0.072	2.740	100	59.6	0.179	0.099	1.810
$\lambda=0.625\,\mu$	109.9	50	0.200	0.072	2.780	100	59.3	0.179	0.097	1.826
$\lambda=0.650\,\mu$	111.1	50	0.203	0.072	2.820	100	56.2	0.179	0.091	1.970
$\lambda=0.675\mu$	116.5	50	0.215	0.072	2.980					

Wave-Lengths.	(n) $V=87.0$; $I=1.00\pm0.02$; $P=7.4$ cm. Drum Readings of Micrometer.		Slit Widths in mm.		Ratio of Intensities.	(o) $V=87.0$; $I=1.10\pm0.02$; $P=7.4$ cm. Drum Readings of Micrometer.		Slit Widths in mm.		Ratio of Intensities.	(p) $V=86.1$; $I=1.20\pm0.02$; $P=7.4$ cm. Drum Readings of Micrometer.		Slit Widths in mm.		Ratio of Intensities.
	S_1	S_n	S_1	S_n		S_1	S_n	S_1	S_n		S_1	S_n	S_1	S_n	
$\lambda=0.425\,\mu$	117.7	80	0.226	0.140	1.613	118.4	70	0.219	0.120	1.828					
$\lambda=0.450\,\mu$	126.0	80	0.235	0.140	1.680	123.0	70	0.232	0.120	1.935	200	94.0	0.394	0.171	2.300
												94.4	0.394	0.172	2.290
$\lambda=0.475\,\mu$	135.6	80	0.255	0.140	1.821	128.8	70	0.241	0.120	2.010	200	92.0	0.394	0.167	2.360
$\lambda=0.500\,\mu$	136.1	80	0.257	0.140	1.835	129.1	70	0.247	0.120	2.060	200	89.6	0.394	0.1595	2.470
$\lambda=0.550\,\mu$	152.3	80	0.293	0.140	2.092	144.8	70	0.275	0.120	2.290	200	82.8	0.394	0.146	2.700
$\lambda=0.600\,\mu$	163.9	80	0.317	0.140	2.260	149.3	70	0.280	0.120	2.330	200	80.4	0.394	0.142	2.780
$\lambda=0.625\,\mu$	169.8	80	0.329	0.140	2.344	162.5	70	0.315	0.120	2.620	200	77.6	0.394	0.135	2.920
$\lambda=0.650\,\mu$	183.0	80	0.356	0.140	2.540	170.1	70	0.330	0.120	2.744	200	75.7	0.394	0.131	3.000
$\lambda=0.675\,\mu$	195.2	80	0.384	0.140	2.740	179.6	70	0.350	0.120	2.920	200	60.14	0.394	0.105	3.080
	(q) $V=85.05$; $I=1.30\pm0.02$; $P=7.4$ cm.					(r) $V=84.6$; $I=1.40\pm0.02$; $P=7.4$ cm.					(s) $V=84.0$; $I=1.50\pm0.02$; $P=7.4$ cm.				
$\lambda=0.425\,\mu$						200	87.4	0.394	0.154	2.560					
$\lambda=0.450\,\mu$	200	90.4	0.394	0.164	2.400	200	85.9	0.394	0.151	2.610	200	83.3	0.394	0.148	2.660
$\lambda=0.475\,\mu$	200	87.9	0.394	0.158	2.495	200	83.9	0.394	0.148	2.661					
$\lambda=0.500\,\mu$	200	85.3	0.394	0.152	2.592	200	80.6	0.394	0.143	2.758	200	78.5	0.394	0.137	2.880
$\lambda=0.550\,\mu$	200	79.6	0.394	0.140	2.818	200	77.0	0.394	0.134	2.940	200	77.3	0.394	0.135	2.920
$\lambda=0.600\,\mu$	200	76.9	0.394	0.134	2.940	200	75.5	0.394	0.130	3.030	200	74.5	0.394	0.130	3.030
$\lambda=0.625\,\mu$	200	76.5	0.394	0.133	2.960	200	74.0	0.394	0.127	3.100					
$\lambda=0.650\,\mu$	200	73.1	0.394	0.128	3.080	200	71.9	0.394	0.122	3.230	200	72.7	0.394	0.125	3.155
$\lambda=0.675\,\mu$	200	70.1	0.394	0.121	3.260	200	70.0	0.394	0.120	3.280					

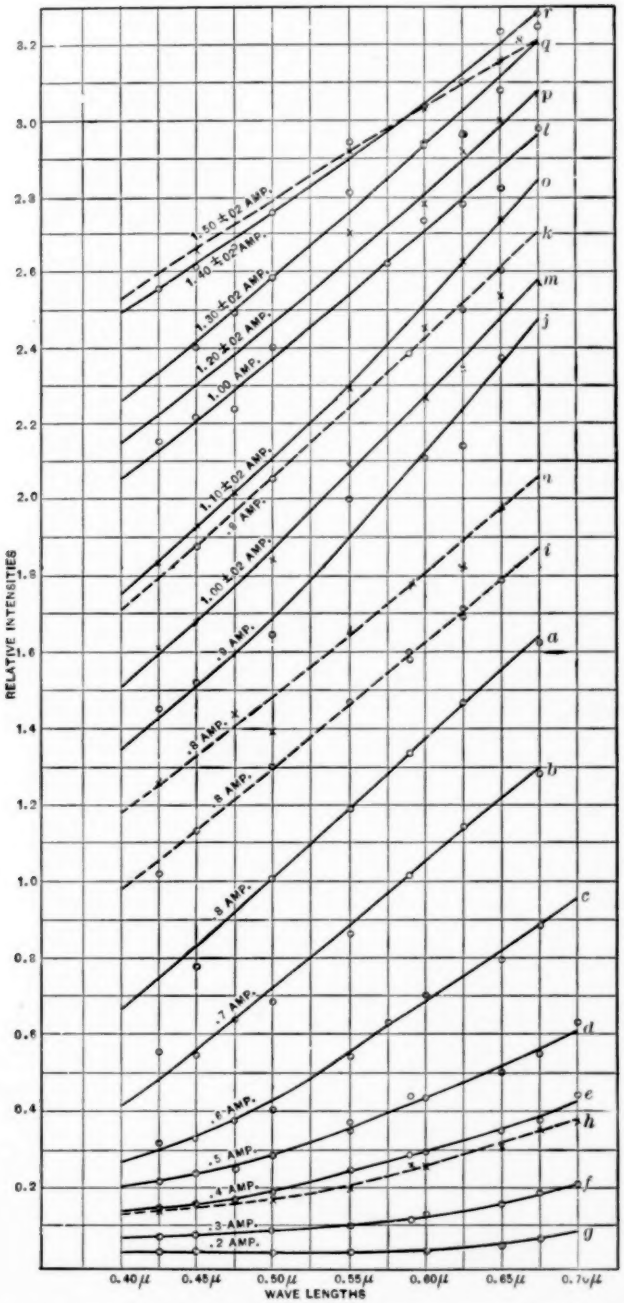

Fig. 9. Comparison of the Nernst Lamp at various current densities with Standard No. 1.

At 1.20 amperes the ballast of the lamp burned out after which the leads to the lamp were connected directly to the terminals of the glower and observations were continued with increasing current strength for each successive set until the maximum was reached. In all about 2,500 observations are represented in the data given in the tables.

In tabulating the data the following symbols are used: P represents the gas pressure in centimeters of water; V, the potential difference in volts between the terminals of the glower; S_1, the slit width of the collimator for standard No. 1; S_2, the slit width of the collimator for standard No. 2; S_H, the slit width of the collimator for the Hefner lamp; S_n, the slit width of the collimator for the Nernst lamp, and (a), (b), (c), $\cdots$, etc., will represent the order of the observations.

At this point it may be noted that the writer is familiar with the work done by Murphy[1] and Capps[2] on the correction for slit widths necessary in spectrophotometric work. No data for this correction with such an instrument as was used here, however, are available. In the second place, correction for slit widths is not so essential here because in the method of using the data given in the tables, the error due to this cause is eliminated, or at least reduced to a minimum. In obtaining the curve showing the relation between the intensities of the Hefner-Altneck lamp and standard No. 1 for waves of different lengths, two curves on different days were carefully determined, and the mean values of the ordinates of these curves were taken and plotted as the ordinates of the curve in Fig. 5. The two curves mentioned are shown in the upper part of Fig. 5 and the data therefor are given in Table I. Thus the intensity in candle power of standard No. 1 for different wave-lengths is determined. The values obtained in the comparison of standard No. 1 with standard No. 2 were treated in a similar manner and the curve, as shown in Fig. 6, obtained. If now the values of the ordinates of the curve in Fig. 6 be multiplied by the corresponding ordinates of the curve in Fig. 5, and if these new values thus obtained be plotted as ordinates with the corresponding wave-lengths as abscissæ, the curve shown in Fig. 8 will be obtained. This

[1] Murphy, Astrophys. Jour., 6, 1, 1897.
[2] Capps, Ibid., 11, 25, 1900.

curve shows the relation existing between the intensity of the acetylene flame used as a standard unit of comparison and the intensity of the Hefner lamp when the influence of the totally reflecting prism is eliminated. This curve agrees with results previously obtained.[1] It is to be noted here that the Hefner lamp was 10 cm. from the collimator slit while the acetylene standard No. 2 was 50 cm. from the slits. Their relative positions are indicated in Fig. 4.

If the values of the ratios of intensities given in Table III. are plotted as ordinates and corresponding wave-lengths as abscissæ, the curves shown in Fig. 9 are obtained. If the ordinates of these curves are divided by one fourth of the corresponding ordinates of Fig. 6 and the values thus obtained are plotted as ordinates with wave-lengths as abscissæ, a series of curves will be obtained showing the relation between the intensities of the Nernst lamp glower at different current strengths and standard No. 2, when both are at a distance of 100 cm. from the collimator slits. These curves are shown in Fig. 10. In these curves the absorption of the totally reflecting prism has been eliminated and the error due to slit widths reduced to a minimum. From an inspection of these curves it is obvious that until a current strength of one ampere is reached the increase in intensity of the glower as compared with acetylene is more rapid in the long waves and less rapid in the short ones. Above this value of the current, the increase in intensity in the region of the short waves is more rapid and there is an apparent decrease in intensity in the region of the long waves beyond 0.600 μ.

If the ordinates of the curve in Fig. 9 be multiplied by the corresponding ordinates of the curve in Fig. 5 and if the values thus obtained be plotted as ordinates with corresponding wave-lengths as abscissæ, the curves shown in Fig. 11 will be obtained. These curves show the relation, wave-length by wave-length, between the relative intensities of the Hefner lamp when it is placed 10 cm. in front of the collimator slits and the Nernst lamp when the latter is placed 100 cm. in front of the slits. These curves confirm the conclusion deduced from the curves in Fig. 10, showing that with a current above 0.90 or 1.00 ampere there is a marked increase in

[1] Hartman, PHYS. REV., 9, p. 185, 1899.

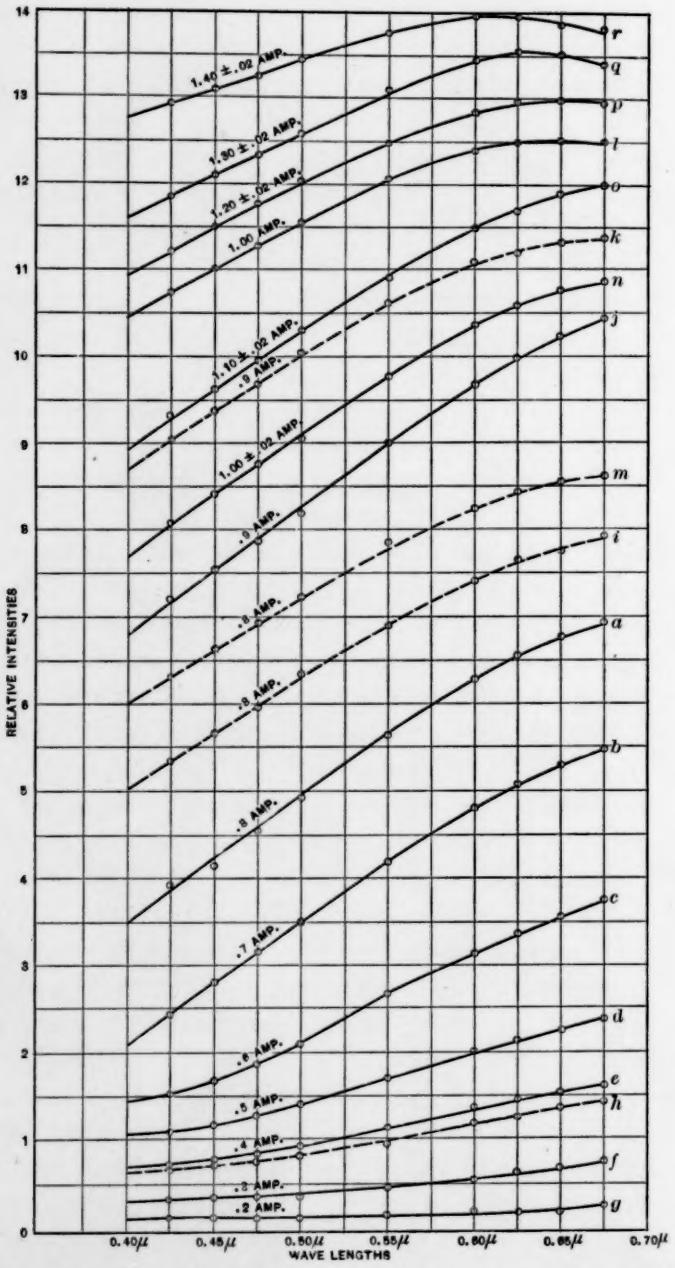

Fig. 10. Comparison of the Nernst lamp at various current densities with standard No. 2. The curves are corrected for absorption and slit widths. Intensities compared with both sources 50 cm. from the collimator slit.

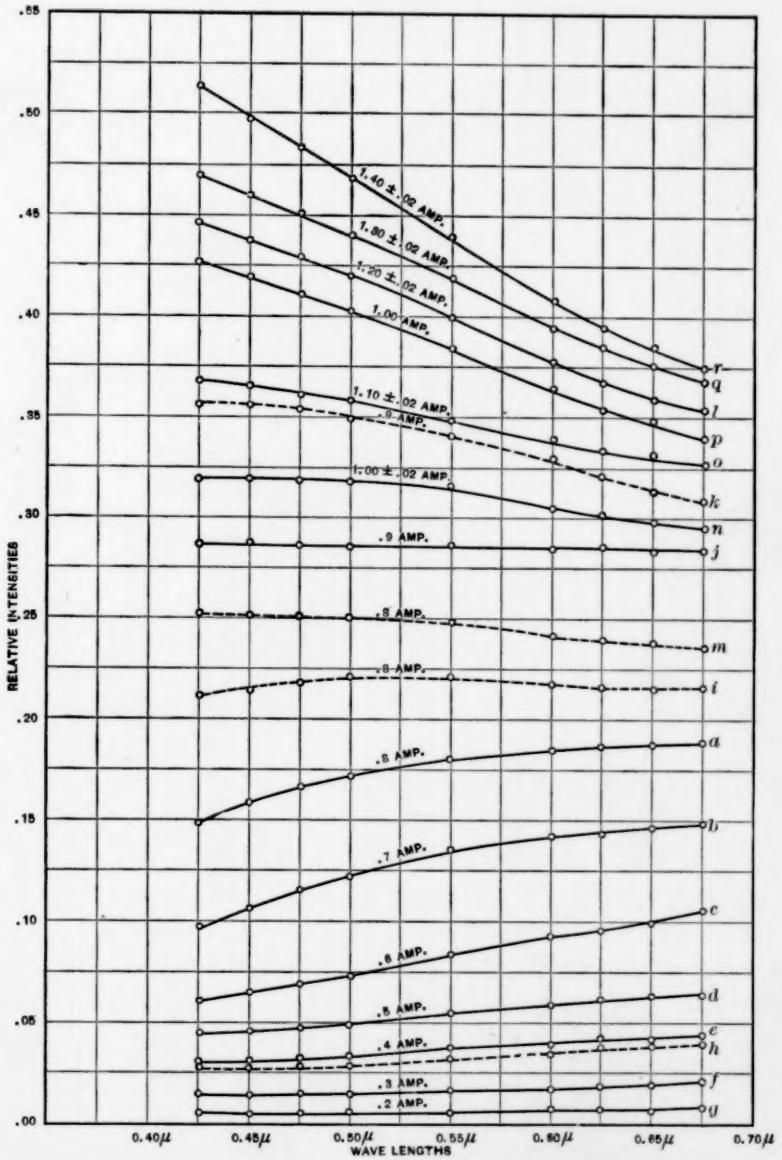

Fig. 11. Comparison of the Nernst lamp with the Hefner lamp.

intensity in the region of the short waves and a corresponding decrease in the intensity of the long waves. Below this value of the current the opposite is true, the increase in intensity being most rapid in the long waves.

It will be noted that a number of the curves are drawn in dotted lines and that these dotted curves bear the same values of current strength as some one of the full line curves beneath them. To this statement there is but one exception, in which case the difference is so small that it may reasonably be ascribed to errors in observation. In the other cases this explanation will not suffice. It will be noted, however, that these dotted curves were obtained at a date later than that on which the full line curves of the same current strength were obtained. Apparently, therefore, this indicates an increase in the intensity of the light emitted from the glower as it ages, even though the current be kept constant. One would expect this if the glower gradually wastes away with continued use and thus decreases in cross-section, for under these conditions a given current would make the glower corresponding hotter the smaller it becomes. There is, however, a probable limitation to this.

In the case of the curves in which the current was furnished by the department alternator, the disturbing factors were under such complete control that the current was kept practically constant. The maximum voltage of the alternator, however, could not be raised to a value high enough to send through the glower a current above 1.00 ampere. After this value was reached, the city 208-volt A. C. mains were used. The conditions were now more unsteady and the needle of the ammeter vibrated through a range of 0.04 amperes and the current values are so indicated. Thus 1.00 ± 0.02 amp. means that the current fluctuated between 0.98 and 1.02 amperes. The reason for the decrease in intensity when current taken from this source was used is not evident to the writer; in both cases current was furnished by a 60-cycle machine.

It will be interesting to compare the luminous radiation from the glower of the Nernst lamp with the radiation from a number of other luminous sources which have been previously studied. To do this the radiation from the acetylene flame is again taken as the standard of comparison, and its relative intensity throughout the

spectrum is arbitrarily taken as unity. In the other sources considered that portion of the spectrum in the region of the *D* line is taken as unity. Plotting wave-lengths as abscissæ and relative intensities as ordinates the series of curves shown in Fig. 12 is ob-

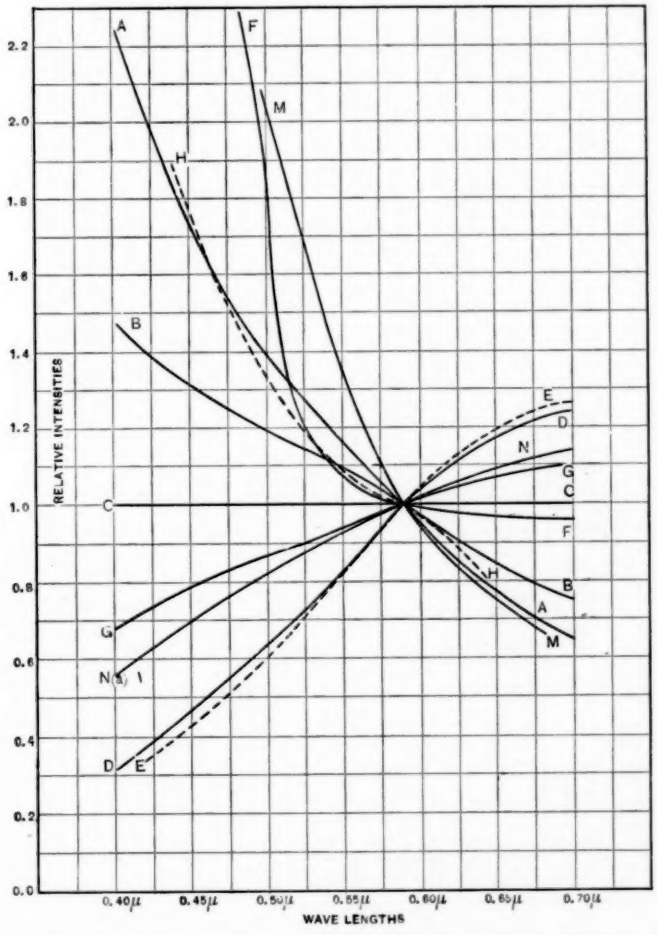

Fig. 12. Color of (*A*) acetylene in oxygen, (*B*) acetylene-hydrogen in oxygen, (*C*) acetylene in air, (*D*) kerosene lamp, (*E*) ordinary gas flame, (*F*) fresh lime light, (*G*) old lime light, (*H*) arc light, (*M*) magnesium light and (N_a) Nernst lamp. Standard, an acetylene flame.

tained. It will be observed that the curve for the Nernst lamp glower, which was taken for the rated normal current strength of 0.80 ampere corresponds most nearly to the radiation from old lime.

The data for many of these curves have been taken from a paper by Professor E. L. Nichols.[1]

In order to have a basis from which to start, it has been arbitrarily assumed in Fig. 13 that the values given by Abney for the luminosity curve of the normal eye[2] would give the luminosity curve of the acetylene flame. The luminosity curves of the Nernst

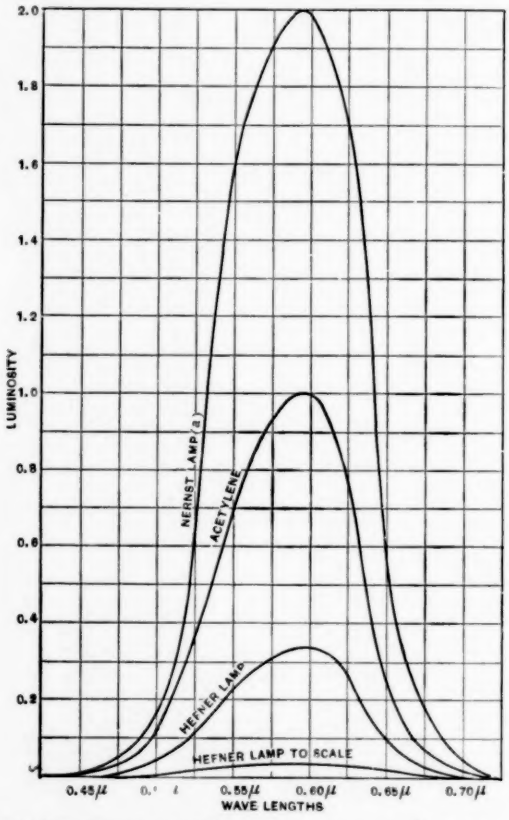

Fig. 13. Luminosity curves of acetylene, Nernst lamp, and Hefner lamp. Abney's normal luminosity curve assumed for acetylene.

lamp and the Hefner lamp were then plotted on the same sheet with this curve. The luminosity of the spectrum of acetylene is taken as unity in the region of the *D* line. In this figure the Nernst lamp curve is reduced to one third its actual size and the curve of the Hefner lamp is magnified ten times. The lower curve repre-

[1] Nichols, Jour. Frank. Inst., **150**, p. 356, 1900.

[2] Abney, Color Vision, p. 211, 1895.

sents the actual luminosity curve of the latter as compared with the acetylene standard No. 2. The luminosity of the Nernst lamp is therefore much greater than that of the acetylene flame.

Since preparing the above, through the courtesy of Dr. Frank Allen, of Cornell University, a luminosity curve of the acetylene flame has been obtained. The luminosity curves of the Nernst and Hefner lamps as compared with acetylene, using this luminosity curve as a basis, are plotted, as described above, in Fig. 14.

To summarize, it appears that at the rated normal current strength the radiation from the glower of the Nernst lamp is relatively rich in the more luminous waves of the spectrum; it exhibits no marked selective radiation; its light intensity apparently increases with age; its light is relatively less rich in the short, but richer in the long waves than that from the acetylene flame, but its radiation rapidly becomes more like that from burning acetylene with increasing current strength; at current strength below the rated normal value, the increase in intensity with increasing current strength is most marked in the long waves, and above this rated normal value of current strength the increase in intensity is most rapid in the short waves; at current strengths below the rated normal value the Nernst lamp is relatively less rich in short waves than the flame of the Hefner lamp, while above this current strength the opposite is true; at rated normal current strength, or slightly above this, the color of the Nernst lamp radiation is very similar to that of the Hefner lamp. The luminous radiation from the Nernst lamp corresponds very closely to that from incandescent old lime.

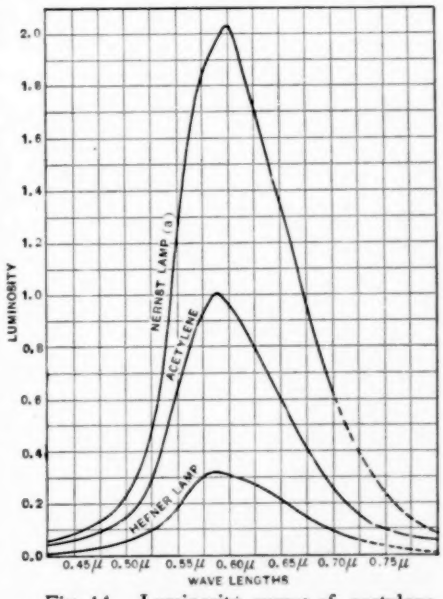

Fig. 14. Luminosity curves of acetylene, Nernst lamp, and Hefner lamp. Luminosity of the acetylene flame based on Allen's curve.

The apparatus used in this investigation has been generously placed at my disposal by Professor Arthur W. Goodspeed, of the Department of Physics, and it is with a sense of gratitude and pleasure that I acknowledge all that he has done to satisfy my every need and thus make this investigation a success. In conclusion, therefore, I take pleasure in thanking him, as well as Dr. Horace C. Richards, for the suggestions they have made and the courtesies they have shown in the course of this investigation.

RANDAL MORGAN LABORATORY OF PHYSICS,
UNIVERSITY OF PENNSYLVANIA, April 10, 1903.

THE PRESSURE DUE TO RADIATION.[1]

(SECOND PAPER.)

BY E. F. NICHOLS AND G. F. HULL.

THE disc was calibrated for temperature in terms of the deflection for a definite sensitiveness of the galvanometer G_1. For this purpose the disc was immersed in a kerosene bath and the galvanometer deflection measured for two different temperatures of the disc. One of these was about 18° C. above the comparatively steady temperature of the room, or calorimeter containing the standard temperature junctions (see Fig. 6), and the other about the same number of degrees below the room temperature. These two temperatures were measured by a Fuess standard thermometer divided into tenths of a degree and calibrated at the Reichsanstalt. Two calibrations of the silver disc were made some days apart. One of these series appears in full in Table V. The first three columns of the table give the zero, direct and reversed reading of the galvanometer G_1. The fourth column gives the temperature of the bath in which the disc was immersed, and the fifth, that of the constant temperature calorimeter. The sixth column gives the deflections of G_1. The seventh column the means of the alternate deflections. The eighth, the mean of the two columns preceding it. The last column gives the difference in temperature between the two calorimeters in degrees C. For the total temperature range in the table, 39.11°, the deflection of G_1 was 393.8 scale divisions for a sensitiveness of $G_1 = 996$. A range of one degree would thus give a deflection of 10.03 divisions for a sensitiveness of $G_1 = 1{,}000$. The mean of two separate calibrations was 9.96 scale divisions for one degree temperature difference.

Before beginning a series of intensity measurements the disc was suspended in an air-chamber containing phosphoric anhydride and

[1] Concluded from July number.

TABLE V.

Calibration of Silver Disc.

Cold Bath.

G_1 Readings.			Disc $T_1°$	Room cal. $T_2°$	Deflection of G_1	Means of Alternate Deflections.	G_1 Means.	$T_2°-T_1°$
Rev.	Zero.	Direct.						
	221.2							
	221.0	35.2	1°.58	20°.05	185.8			
	220.9							
402.0	**221.0**		1°.60	20°.07	181.0	185.7	183.4	18.47
	221.2							
	221.2	35.7	1°.57	20°.10	185.5	181.3	183.4	18.53
	221.2							
403.1	**221.5**		1°.56	20°.14	181.6	185.8	183.7	18.58
	221.9							
	222.0	35.9	1°.54	20°.16	186.1	182.2	184.1	18.62
	222.1							
405.0	**222.2**		1°.52	20°.20	182.8	186.3	184.5	18.68
	222.4							
	222.7	36.2	1°.57	20°.22	186.5	182.7	184.6	18.65
	223.0							
405.8	**223.1**		1°.60	20°.26	182.7	186.7	184.7	18.66
	223.2							
	223.3	36.3	1°.59	20°.30	187.0			
	223.5						184.0	18°.60

Correction to $T_1 = 0°.00$.

Warm Bath.

	217.3							
	218.6	434.2	41°.45	20°.40	215.6			
	219.9							
2.0	**220.5**		41°.35	20°.42	218.5	213.7	216.1	20.93
	221.1							
	222.4	434.2	41°.25	20°.44	211.8	216.4	214.1	20.81
	223.7							
10.0	**224.4**		41°.08	20°.50	214.4	209.1	211.7	20.58
	225.1							
	225.7	432.1	40°.90	20°.55	206.4	212.4	209.4	20.35
	226.3							
16.3	**226.7**		40°.80	20°.60	210.4	204.8	207.6	20.20
	227.2							
	227.8	431.0	40°.67	20°.61	203.2	208.5	205.8	20.07
	228.4							
21.8	**228.5**		40°.55	20°.63	206.7	201.8	204.2	19.92
	228.7							
	229.0	429.4	40°.43	20°.65	200.4		209.8	20°.41
	229.3							

Correction to $T_1 = 0°.10$. Corrected, 20.51

surrounded by a jacket of ice and salt. The disc was thus lowered to a temperature of about zero degrees and was then quickly transferred to the chamber C (Fig. 6), and the beam was directed upon it. When its temperature had risen to within five or six degrees of that of the chamber C, galvanometer readings were made at intervals of five seconds until the disc was heated to a temperature several degrees above its surroundings. The temperature of the chamber C was determined by removing the disc and cooling it to a point near the room temperature, then replacing it and observing its rate of temperature change for several minutes.

The note-book record of one series of observations showing the heating of the disc by the light beam is given in full in Table VI. It will be seen from the table that the temperature of the disc passed

TABLE VI.

August 16. *Energy Measurements. Through Air. Series* 4.
Zero of Galvanometer G_1 (*closed circuit*) *Determined by Method of Cooling* = 216.8 = *Reading at Room Temperature.*

Time.	G_1	Time.	G_1	ΔG_1	Δt	$\frac{\Delta G_1}{\Delta t}$ (in mm. per sec.)
0 secs.	174.5	60 secs.	253.2	78.7	60 secs.	1.312
5 "	182.0	55 "	247.3	65.3	50 "	1.306
10 "	189.0	50 "	241.3	52.3	40 "	1.308
15 "	196.2	45 "	235.2	39.0	30 "	1.300
20 "	203.0	40 "	229.1	26.1	20 "	1.305
25 "	209.7	35 "	222.8	13.1	10 "	1.310
30 "	216.4				Average	1.307

The lamp reading (G_2) was 924.
The sensitiveness of G_2 was 667, and of G_1 was 996.
$\frac{\Delta G_1}{\Delta t}$ reduced to standard conditions becomes

$$1.307 \times 667 \times 996 \div (924 \times 1{,}000) = 0.943 \text{ mm. per sec.}$$

that of the chamber thirty seconds after the beginning of the series. The readings of G_1 at equal time intervals on either side of the zero are on horizontal lines. The last column of the table contains the rate at which the galvanometer deflection was changing when the disc and its surroundings were at the same temperature.

Energy series were made "through air," "through red glass," and "through water cell," as in the pressure measurements. During the experiment the black coatings were frequently cleaned off

from the disc and new ones deposited. The final result therefore does not correspond to an individual, but to an average coating.

To correct for any inequality between the two disc thermo-junctions or any lack of symmetry in their positions, referred to the central plane of the disc, which might prevent the mean temperature of the two junctions from representing the mean temperature of the mass, series of observations were made on each face of the disc. The

TABLE VII.

Front Face.

Date.	Through Air.					Through Red Glass.			Through Water Cell.		
	$\frac{\delta G_1}{\delta t}$	G_2 (Lamp).	S_1	S_2	$\frac{\delta G_1}{\delta t}$ Reduced to Standard	$\frac{\delta G_1}{\delta t}$	G_2	$\frac{\delta G_1}{\delta t}$ Reduced to Standard	$\frac{\delta G_1}{\delta t}$	G_2	$\frac{\delta G_1}{\delta t}$ Reduced to Standard
Aug. 10	1.387	980	990	689	.965				.437	345	.864
" "	1.263	920	990	689	.936				.400	311	.877
" "									.369	279	.902
" 11	1.244	866	986	701	.992	.750	546	.950	.412	315	.905
" "	1.455	1010	986	701	.995	.750	546	.950	.510	382	.922
" "	1.505	1047	986	701	.994				.516	381	.935
" 16	1.447	1022	996	669	.942	.736	529	.927	.416	327	.873
" "	1.284	886	996	669	.966	.740	527	.936	.451	352	.853
" "	1.316	925	996	669	.948	.797	550	.965	.502	382	.875
" "	1.307	924	996	669	.943						
" 18	1.598	1110	995	667	.955	.738	515	.952	.449	333	.895
" "	1.550	1047	995	667	.984	.732	518	.940	.445	342	.865
" "	1.548	1031	995	667	.995	.730	518	.938	.451	346	.867
" "	1.410	957	995	667	.977						
" "	1.330	898	995	667	.983						
" 19	1.241	862	1001	675	.975	.760	532	.965	.451	343	.892
" "	1.360	934	1001	675	.985	.728	512	.960	.452	338	.904
" "	1.324	905	1001	675	.990	.738	525	.950	.466	351	.898
" "	1.364	934	1001	675	.988						
					Average 0.973 ± 0.003			Average 0.948 ± 0.002			Average 0.888 ± 0.004

black coating was always cleaned off from the face of the disc away from the light. All of the series of energy measurements are gathered together in Tables VII. and VIII. In the tables, under the head "through air," the first column contains the observed rate of increase in the galvanometer deflection G_1, when the disc and its surroundings were at the same temperature; the second column,

TABLE VIII.

Rear Face.

Date.	Through Air.					Through Red Glass.			Through Water Cell.		
	$\frac{\delta G_1}{\delta t}$	G_2 (Lamp).	S_1	S_2	$\frac{\delta G_1}{\delta t}$ Reduced to Standard.	$\frac{\delta G_1}{\delta t}$	G_2	$\frac{\delta G_1}{\delta t}$ Reduced to Standard.	$\frac{\delta G_1}{\delta t}$	G_2	$\frac{\delta G_1}{\delta t}$ Reduced to Standard
Aug. 12.	1.374	960	991	684	.970	.808	578	.949	.495	370	.906
" 12.	1.331	932	991	684	.968	.740	536	.935	.434	320	.919
" 12.	1.284	900	991	684	.967	.765	542	.957	.489	371	.895
" 15.	1.428	992	996	670	.960	.703	506	.926	.490	368	.890
" 15.	1.428	984	996	670	.968	.742	526	.941	.466	352	.885
" 15.	1.531	1068	996	670	.962	.765	551	.926	.440	337	.873
" 20.	1.477	1047	996	685	.961	.703	522	.918	.458	375	.833
" 20.	1.520	1090	996	685	.951	.760	537	.965	.497	400	.848
" 20.	1.576	1130	996	685	.951	.781	570	.935	.507	408	.848
" 20.	1.568	1124	996	685	.950						
" 21.	1.783	1224	995	668	.970	.846	604	.932	.503	393	.852
" 21.	1.773	1232	995	668	.957	.790	575	.915	.481	377	.850
" 21.	1.705	1190	995	668	.953	.803	575	.930	.483	373	.862
" 21.	1.452	1019	995	668	.948						
				Average	0.960 ± 0.0014			Average 0.936 ± 0.003			Average 0.872 ± 0.005
Average of front and rear face, 0.966 ± 0.0034								0.942 ± 0.0036			0.880 ± 0.0064

the corresponding mean lamp deflections of galvanometer G_2. The third and fourth columns contain the sensitiveness of galvanometers G_1 and G_2, respectively, and the last column the values of the first column reduced to standard lamp and standard sensitiveness of both instruments. The series on the two faces of the disc are recorded and averaged separately, then combined with their probable errors in the general average at the end of Table VIII.

Tables VII. and VIII. give the following results: The average increase in the reading of G_1 for standard conditions is 0.966 mm. per second. From the thermal calibration, a deflection of 9.96 divisions corresponds to a temperature difference of 1° C. Consequently the rise in temperature of the silver disc per second when the light passed:

(*a*) through air = 0.966 ÷ 9.96 = (0°.0970 ± 0°.00034 C.).

(*b*) through red glass = 0.942 ÷ 9.96 = (0°.0946 ± 0°.00036 C.).

(c) through water cell = 0.880 ÷ 9.96 = (0°.0884 ± 0°.00064 C.).

The mass of the silver disc was 4.80 grams, its specific heat[1] at 18°C. = 0.0556; the mechanical equivalent of heat at 18° C. = 4.272 × 10^7 ergs.[2] Consequently the energy of the standard radiation is

(a) through air, 0.0970 × 4.80 × 0.0556 × 4.272 × 10^7
or $E_a = (1.108 \pm 0.004) \times 10^6$ ergs per second.

(b) through red glass, $E_g = (1.078 \pm 0.004) \times 10^6$ " " "

(c) through water cell, $E_w = (1.008 \pm 0.007) \times 10^6$ " " "

REFLECTING POWER OF THE SURFACES USED.

According to Maxwell and Bartoli, the pressure in dynes per square centimeter for normal incidence is equal to the energy in ergs in unit volume of the medium. The energy in unit volume is made up of both the direct and reflected beams. If E is the intensity of the incident beam and ρ the reflection coefficient, the pressure $p = \frac{E(1+\rho)}{V}$, where V is the velocity of light. The methods for measuring ρ and E have already been described. The determination of ρ for both sides of the vanes C and D was made as follows. The supports of the torsion balance were replaecd by the divided circular plate A (Fig. 7), of a force table which could be rotated about a central, vertical axis. The rod about which the plate turned passed up through the plate and at its top the mirror holder bb was fastened. The vanes were freshly silvered and mounted on a plate glass carrier aa, which was held by a clamp against the back face of bb. The beam was directed on the vanes by the lens L_3 (Figs. 3 and 7) exactly as it had been in the pressure observations. After reflection from the vane the beam fell on a concave mirror M which projected an image of the vane upon a simple sheet bolometer B, forming the unknown resistance of a post-office-box bridge. The current was supplied from storage cells and the galvanometer was the same as used in the energy determinations, but fitted with low resistance coils. The bolometer was covered by the

[1] U. Behn, Ann. Phys., IV., p. 266, 1900.

[2] Mean of Rowland's and Griffith's values, Phil. Trans., V., pp. 184, 496, 1893.

bell-jar used earlier. The mirror M, the bell-jar and bolometer were attached to the plate of the force table. The full line diagram shows the arrangement for reflection. The dotted figure shows the position for a measurement of the direct beam. All measurements of direct reflection were made for an angle of incidence of 12°.5.

The method of observing will be seen from the note-book record of a single series of measurements given in Table IX. In the table,

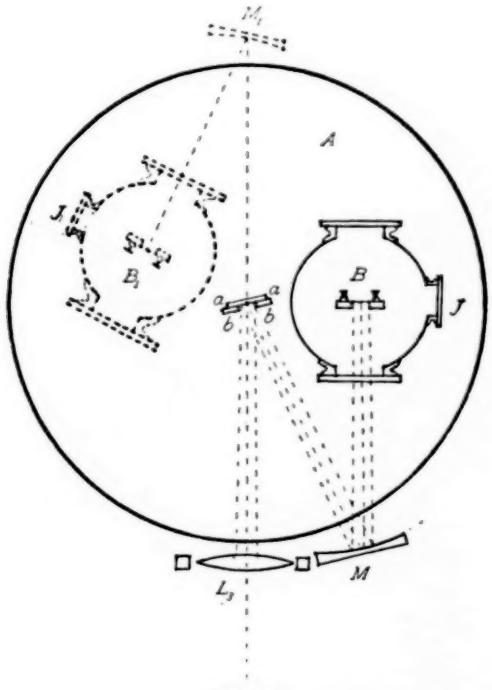

Fig. 7.

D and R indicate direct and reflected beams, respectively. The first and second columns contain the zero points and end of swings of the galvanometer G_1, and the third column, the deflection. The remaining columns, in order, contain the lamp galvanometer deflection ; the deflection of G_1 reduced to constant lamp ; the means of each pair of D or R values ; the means of alternate readings ; and the final column, the quotients of the two preceding columns which are the reflection coefficients sought. In all, three series of measurements were made on the silver, and two series on the glass-silver

TABLE IX.

October 31, 1902. Reflection Coefficient of D_g. Air.

G_1 Zero.	G_1 Turning Point.	Deflect G_1	Lamp.	G_1 Reduced to Standard.	Averages.	Alternate Averages.	Reflection Coefficient.
R 350.0	159.5	190.5	132.6	143.8			
349.0	152.0	197.0	139.3	141.3	142.5		
D 349.5	100.5	249.0	136.8	182.1			
350.0	111.5	238.5	130.5	183.0	182.5	142.0	.779
R 346.0	177.0	169.0	119.3	141.2			
347.0	171.0	176.0	124.4	141.7	141.5	182.5	.775
D 348.5	123.0	225.5	124.0	181.8			
348.5	120.0	228.5	125.0	183.0	182.4	141.1	.773
R 345.0	172.0	173.0	122.6	141.0			
345.0	171.0	174.0	124.0	140.3	140.6	182.7	.770
D 346.0	132.0	214.0	115.5?				
346.0	124.0	221.0	120.7	183.0	183.0	141.2	.773
R 344.0	173.0	171.0	120.7	141.8			
344.5	171.0	173.5	122.6	141.6	141.7	182.1	.778
D 346.0	119.0	227.0	125.3	181.0			
346.0	117.5	228.5	126.0	181.3	181.2	141.6	.781
R 342.0	174.0	168.0	118.0	142.3			
342.0	170.5	171.5	122.0	140.8	141.5	181.8	.780
D 347.0	130.0	217.0	119.0	182.3			
347.0	134.0	213.0	116.7	182.7	182.5	141.4	.775
R 341.5	174.5	167.0	118.0	141.3			
341.0	173.0	168.0	119.0	141.1	141.2		
						Average	0.776

faces of each vane. To get average coefficients which would represent the range of condition of the mirrors during the pressure measurements, the vanes were cleaned and new silver coatings deposited between each two series on the same vane. The reflection coefficients are collected in Table X. For each surface studied the diffused reflection for a beam which had traversed air was determined by setting the mirror holder for normal incidence. The diffuse energy reflected at an angle of 25° falling on the full aperture of the mirror M was measured, and the total diffuse energy for the hemisphere computed on the basis of the cosine law. If $I_\theta \partial A$ is the amount of diffuse radiation falling normally upon the area ∂A, distant r from the vane and at an angle θ with the incident radiation, then $I_\theta \partial A = I_0 \cos\theta \, \partial A$. The total amount of diffuse radiation $= \int\int I_0 \cos\theta \partial A$,

over the surface of the hemisphere $= \int_0^{\frac{\pi}{2}} 2\pi r^2 \ I_0 \cos\theta \sin\theta \partial\theta$ $= \pi I_0 r^2$. This integral is the amount of the diffuse radiation in Table X. The force, due to radiation of intensity $I_\theta \partial A$, normal to the vein is $I_\theta \cos\theta \partial A$, and the total is equal to $\int_0^{\frac{\pi}{2}} 2\pi r^2 I_0 \cos^2\theta \sin\theta \partial\theta = \frac{2}{3}\pi I_0^2$. It is thus seen that of the diffuse reflection, two

TABLE X.

Reflection Coefficients in Percentages.

	C_a.				C_g.			
	Through Air.	Red Glass.	Water.	Diffuse.	Through Air.	Red Glass.	Water.	Diffuse.
	92.8	94.5	88.9	0.98	77.8	75.9	80.8	
	89.8	90.8	86.0	0.92	77.6	76.6	80.0	1.6
	90.8			1.23				
Average	91.1	92.7	87.5	1.04	77.7	76.3	80.4	1.6
	D_a.				D_g.			
	95.0	96.3	91.5	2.2	77.6	76.5	81.0	2.8
	92.0	94.0	90.4		76.7	75.2	79.7	2.2
	94.8	95.0	92.3	0.8				
Average	93.9	95.1	91.4	1.5	77.2	75.9	80.4	2.5

Average Reflection.

	Air-Silver.				Glass-Silver.			
	92.5	93.9	89.5	1.3	77.5	76.1	80.4	2.0

Corrected Reflection Coefficients.

	92.0	93.4	89.0		77.6	76.2	80.5	

Average Coefficients through Air, *84.8*; Red Glass, *84.8*; Water, *84.8*.

thirds is effective as light pressure. This increases the air-silver reflection coefficients by 0.9 per cent. and the glass-silver values by 1.3 per cent. The small glass rod *d* (Fig. 2), not present in the reflection measurements, decreased the reflecting area of the silvered surfaces in the pressure measurements by 1.54 per cent. The air-silver values are thus decreased by $0.92 \times 1.54 = 1.4$ per cent., and the glass-silver values by $0.78 \times 1.54 = 1.2$ per cent. The application of these two corrections gives the final corrected coefficients

in Table X. The diffuse reflection of black coatings deposited by the method used in blackening the silver disc was measured and computed in the same manner as the diffused reflection from the vanes C and D. The agreement found by Angström[1] between the diffuse reflection of matte surfaces for normal incidence, and the cosine law was abundantly close for the present purpose. Five determinations of this reflection were made under different conditions and with different coatings. The values in percentages of the incident beam were 4.4 per cent., 4.5 per cent., 4.2 per cent., 4.6 per cent. and 5.2 per cent.; average, 4.6 per cent. Thus only 95.4 per cent. of the incident beam was absorbed by the black coating on the silver disc in producing the temperature increase observed. Hence the true energy of the beam is equal to the observed energy divided by 0.954.

The silver disc, diameter 13.3 mm., used in the energy measurements, received long waves and scattered radiation which passed round and through the light pressure vanes of diameter 12.8 mm. This amount was experimentally determined for both thin and thick silver coatings in order to approximate to the average condition of the coatings in the light pressure measurements and it was found to average (*a*) through air, 1.40 per cent.; (*b*) through red glass, 1.44 per cent.; (*c*) through water, 0.46 per cent. On this account the energy E of the standard radiation must be reduced by the above percentages.[2] Applying these corrections and the corrections due to the diffused radiation from the black coating on the silver disc, the energy of the standard radiation becomes

(*a*) through air, $E_a \times \frac{0.986}{0.954}$;

(*b*) through red glass, $E_g \times \frac{0.986}{0.954}$;

(*c*) through water, $E_w \times \frac{0.995}{0.954}$.

[1] K. Ångström, Wied. Ann., XXVI., 271, 1885.

[2] As the average pitch of the cone of the incident beam was about one part in forty, no correction need be applied for inclination. Furthermore, the inside of the bell-jar was blackened and the zero of the balance was so chosen that energy reflected from the window admitting the beam could produce no pressure effects.

Hence the pressure produced by standard radiation calculated by Maxwell's formula,

$$p = \frac{E(1+\rho)}{3 \times 10^{10}},$$

since $\rho = 0.848$, becomes

(*a*) through air $$p_a = E_a \times \frac{1.848}{3 \times 10^{10}} \times \frac{0.986}{0.954}$$

$$= 1.108 \times \frac{1.848}{3 \times 10^{10}} \times \frac{0.986}{0.954} \times 10^6 \text{ dynes}$$

$$= (7.05 \pm 0.03) \times 10^{-5} \text{ dynes};$$

(*b*) through red glass $p_g = E_g \times \frac{1.848}{3 \times 10^{10}} \times \frac{0.986}{0.954}$

$$= 1.078 \times \frac{1.848}{3 \times 10^{10}} \times \frac{0.986}{0.954} \times 10^6 \text{ dynes}$$

$$= (6.86 \pm 0.03) \times 10^{-5} \text{ dynes};$$

(*c*) through water $p_w = E_w \times \frac{1.848}{3 \times 10^{10}} \times \frac{0.995}{0.954}$

$$= 1.008 \times \frac{1.848}{3 \times 10^{10}} \times \frac{0.995}{0.954} \times 10^6 \text{ dynes}$$

$$= (6.48 \pm 0.04) \times 10^{-5} \text{ dynes}.$$

A comparison of observed and computed pressures follows:

	Observed Values in 10^{-5} Dynes.	Computed Values in 10^{-5} Dynes	Obs. − comp. in Percentages.
Through air,	$p_a = 7.01 \pm 0.02$[1]	7.05 ± 0.03	-0.6
Through red glass,	$p_g = 6.94 \pm 0.02$	6.86 ± 0.03	$+1.1$
Through water,	$p_w = 6.52 \pm 0.03$	6.48 ± 0.04	-0.6

[1] The pressure and energy measurements for the three different wave groups through air, red glass and water cell constitute three independent experiments. In the values for pressure, 7.01, 6.94, 6.52, equality is not to be looked for. The difference arises from the different reflecting power of the 45° glass plate (Fig. 3) for the different beams and from the fact that the indications of the lamp galvanometer G_2 connected with the bolometer R, were probably not strictly proportional to energy for throws differing as widely as 33, 60 and 100, which, roughly, were the relative intensities of the beams through water cell, red glass and air. The function of the lamp bolometer and galvanometer was purely to keep a check on the small variations of the lamp which rarely fluctuated more than 10 per cent. on either side of the mean value.

An estimate of the approximate magnitude of the gas action not eliminated by the ballistic method of observation, may be reached from the following considerations.

When radiation falls upon a vane of the torsion balance, part of it is absorbed by the silver surface. From the amounts directly and diffusely reflected, as given in Table X., the amount transmitted by the average surface (experimentally determined but not given in Table X.), the effect of the glass rod and the reflection coefficient of the glass surface, it was found that, when the silver side of the vane was toward the radiation source, the absorption coefficient for radiation through air was 6 per cent. and when the glass surface was forward, it was 18 per cent.

The total force acting on the vane is made up of two parts, that due to radiation pressure and that due to gas action. Let F_r be the force due to the first cause, assuming that all the radiation is absorbed, and F_g the effect due to the second, on the same condition. Then the total effect, when the silver side of the vane is forward and the radiation is "through air," is $1.92\ F_r + 0.06\ F_g$. When the glass side is forward the total effect is $1.776\ F_r - 0.18\ F_g$. Making these expressions equal to the reduced deflection (Table III., columns 11 and 12) on the silver and glass surfaces respectively, we have two equations by means of which the values of F_r and F_g may be obtained. Hence the effect due to gas action on each face of the vane is approximately determinate, as is also the part ($0.06F_g$) not eliminated when we average the two columns to obtain column 13.

Applying this method to all the results of Table III. (with the exception of those results taken with poor mirrors as shown by our notes), the gas action present in the ballastic deflections "through air" is 0.8 per cent. Applying the corresponding data and equations to Table IV., the gas action present in the red glass values is 1.1 per cent. and in the water cell values, 0.3 per cent. The sign of F_g comes out negative, which means that the gas action was suction.

This reasoning assumes that the glass faces of the vanes during the six seconds' exposure are not warmed by absorption nor by the conduction of heat through the thin glass from the silver coating.

The effect of any such absorption or conduction would be to diminish the computed gas action. As estimated from the static observations, the gas action in the ballistic measurements is comparable in magnitude with the computed values obtained above, and of the same sign. Both results show that the uneliminated gas action by the most liberal estimate cannot have exceeded one per cent. of the radiation pressure. Because of its smallness and indefiniteness no correction for gas action has been made to the final pressure values. If corrections were applied its effect would be to slightly reduce the observed pressures.

Aside from the measurements of pressure and energy for which the probable errors are given, the percentage accuracies in the other measurements entering into the computations, and their effects upon the final result follow:

1. Quantities which affect individual series:

(*a*) Pressure values —

		Per cent.		Per cent.
Period of balance,	T,	accurate to 0.2;	effect on result	0.0
Lever arm of balance,	l,	" 0.1;	" "	0.0
Constant of galvanometer	G_2,	" 0.5;	" "	0.0
Estimate of possible error due to changing ratio of period of G_2 to length of exposure of bolometer,		" 0.4;	" "	0.1

(*b*) Energy values —

Constant of galvanometer	G_1,	accurate to 0.1;	effect on result	0.0
" "	G_2,	" 0.5;	" "	0.0

2. Quantities which affect final averages:

(*a*) Pressure values —

	Per cent.		Per cent.
Torsion of fiber,	accurate to 0.2;	effect on result	0.2
Reducing factor 1.357,	" 0.1;	" "	0.1
Reducing factor 1.550 for G_2,	" 0.2;	" "	0.2
Reflection of surfaces of vanes,	" 0.4;	" "	0.2

(*b*) Energy values —

Mass of silver disc,	accurate to 0.1;	effect on result	0.1
Thermal calibration of disc,	" 0.5;	" "	0.5
Diffuse reflection black coating,	" 5.0;	" "	0.1

From the agreement, within the probable error, of the air, red glass and water values with the theory, it appears that the radiation pressure depends only upon the intensity of the radiation and is independent of the wave-length.

The Maxwell-Bartoli theory is thus quantitatively confirmed within the probable errors of observation.

To Professor J. L. Mann and Messrs. J. A. Brown, Philip Fox, L. E. Woodman, H. R. Willard, H. E. K. Ruppel, and A. V. Ruggles, the writers are indebted for friendly assistance given at some of the stages in the protracted course of these experiments.

WILDER LABORATORY, DARTMOUTH COLLEGE,
HANOVER, N. H., February, 1903.

THE SPECIFIC HEAT OF SOLUTIONS. IV.

By William Francis Magie.

1. This paper is a continuation of others published under the same title in this Review, Vols. IX., XIII., XIV. It presents the specific heats and the heats of solution of the solids which were used in solution in the previous investigations, and also a speculation regarding a possible distribution of the degrees of freedom among their atoms.

2. *Method of Determining the Specific Heats.*—The calorimeter used in the determination of the specific heats was the form of Pfaundler's calorimeter already described. A liquid, either aniline or kerosene, was chosen as standard, which, according to the tests made with it, did not dissolve the solids. Appropriate amounts of this standard liquid were introduced into the calorimeter cups, such that when the solid was also introduced into the cup containing the smaller amount of liquid, the heat capacity, and therefore the rise of temperature, was about the same in both cups. Ordinarily 30 grams of the solid were used and the desired adjustment could be obtained without further trial by assuming 0.30 as the specific heat of the solid. The solids were introduced in the form of fine powder. Their specific gravities where such that when the liquid was stirred they were carried about with it, and easily and quickly took its temperature. The process of determining the relative heat capacities was carried out as has already been described, and from the results thus obtained the specific heats of the solids were calculated. For some reason which I have not been able to discover, but suspect to be the adhesion of a sheet of liquid to the walls of the cups, the rise and fall of temperature during the preliminary and final readings were not so regular as in the previous experiments with aqueous solutions, and the true initial and final temperatures had to be estimated. The uncertainty thus occasioned rendered the successive

determinations less accordant than those made with aqueous solutions, and the final results may be in error by two or three per cent. Errors may also have arisen if any of the solid was dissolved by the liquid used. These errors, if such there were, would generally have made the result obtained too large. As I carefully examined each substance about which there was any doubt by watching small crystals of it placed in the liquid and left standing for some days, I feel quite confident that no appreciable solution took place. This conclusion is confirmed by the fact that the temperature of the liquid containing the solid showed no evidence of any absorption of heat.

3. *The Standard Liquids.* — The results obtained depend upon the values taken for the specific heat of the standard liquid. When this was aniline, the specific heat was taken to be 0.5155. This is the value given by Griffiths[1] for 20°. The specific heat of the kerosene was obtained by comparison with nitrobenzol, the specific heat of which at 20°, from data given by Regnault,[2] is 0.336. The specific heat for kerosene thus obtained is 0.4573, between 14° and 25°. As a percentage error in the specific heat of the standard liquid introduces the same percentage error in the final result, and as these assumed specific heats are not likely in error by as much as one per cent., they may be taken as correct. As a test of the correctness of the value obtained for kerosene and of the general validity of the method, I determined the specific heat of water by using 11.33 grams of it, introduced into the cup containing 578.13 grams of kerosene, and comparing the heat capacity of the mixture with that of 600 grams of kerosene in the other cup. The specific heats obtained from two observations were 1.023 and 0.976, the mean being 0.9995. The deviations of these results from the mean show what I consider to be the probable range of error in the determinations made by this method.

4. *Results.* — In the following table are collected the results of the measurements. Column I. contains the molecular weights of the substances; Column II., their specific heats; Column III., their molecular heats; Column IV., the molecular heats of the same substances in solution. When one number is given in Column IV.

[1] Phil. Mag., January–June, 1895, p. 47.
[2] Landolt and Börnstein's Tables, p. 337.

the solvent used was water. When two numbers are given the upper one was obtained with water, the lower one with alcohol, as the solvent. The temperature range was from 14° to 26°. The substances marked A were determined in aniline, those marked K, in kerosene. I have added some results obtained by other observers, whose names are given below.

Substance.	I.	II.	III.	IV.
Cane Sugar.	342	0.301	103[1]	152.8
Hydrous milk sugar, A.	360	0.298	107	147
Anhydrous milk sugar, A.	342	0.288	99	147
Maltose, A.	360	0.322	116	142.7
Dextrose, A.	180	0.313	56.3	78.8
Levulose, K.	180	0.276	49.7	89.6
Mannite, A.	182	0.315	57.3	108
Dulcite, K.	182	0.283	51.5	97.5
Resorcin, K.	110	0.266	29.2	63.4
				56.8
Hydroquinone, K.	110	0.258	28.3	63.4
				56.8
Pyrocatechin, K.	110	0.313	34.4	75.5
				56.8
Urea, K.	60	0.321	19.2	21
				28
Glycerine, liquid.	92	0.576	53[2]	54
Phenol, liquid.	94	0.561	52.7[3]	71.5
				51.4

I was not able to make successful observations either with acetamide or with phenol.

5. *Discussion of the Results.*—It appears from the results here presented that the molecular heats of isomers are not always the same. In the case of resorcin and hydroquinone, they probably are the same, the difference being not greater than might be accounted for by experimental error, and the exact equality of the molecular heats of these substances in solution, both in water and in alcohol, rendering a similar equality for the solids not improbable. The other pairs of isomers all show marked differences.

In most cases the apparent molecular heat of the substance is

[1] Kopp, L. and B.'s Tables, p. 330.
[2] Emo, L. and B.'s Tables, p. 338.
[3] Schlamp, Beiblätter, XX., 1896, p. 857.

considerably increased when it goes into solution. Urea is the only solid which exhibits an exception to this rule. The molecular heat for solid urea is possibly too low. It may not change at all, and at any rate it changes but little, when urea is dissolved in water. It is, however, increased by solution in alcohol. Liquid glycerine retains the same molecular heat when dissolved in water. Liquid phenol dissolves in alcohol without change; its molecular heat is increased by solution in water.

6. *Possible Hypotheses Regarding Specific Heats of Solutions.* — In one of my earlier papers on this subject I have given reasons for believing that the dissociated ions of an electrolyte, dissolved in water, are joined with groups of water molecules, whose specific heat is diminished by the combination. The calculation of the heat capacity of an aqueous solution of an electrolyte, on this hypothesis, leads to a term in the formula which represents the molecular heat of the undissociated portion of the electrolyte. This is, in most cases tested, greater than the molecular heat of the solid solute. In a few cases the two are equal. This change in molecular heat may be due to combination with a group of water molecules, whose specific heat is increased by the combination; and a similar combination, leading to a similar change in the molecular heats, may occur in the case of non-electrolytes.

It is, however, also possible that the process of solution, in the case of non-electrolytes, involves a loosening of the bonds which confine the atoms in the molecule, so that the increase in the molecular heat is due to an increase in the degrees of freedom of the atoms. In what follows I shall adopt this hypothesis provisionally, and endeavor to construct with it a consistent scheme showing a possible distribution of degrees of freedom among the atoms of the various substances as solids and in solution.

7. *Staigmüller's Formula.* — A formula has been proposed by Staigmüller,[1] from which it is possible, on plausible suppositions as to the degrees of freedom possessed by the atoms in the molecules of gases, to calculate the specific heats of those gases. The molecules we are now considering are so complicated in structure that no hypothesis as to their degrees of freedom can be made directly.

[1] Wied. Ann., Vol. 65, p. 655.

But if we consider Staigmüller's formula as established by its applicability to gases, we may use it to calculate the degrees of freedom of these molecules, and then see if they can be made to fall into any consistent scheme.

Staigmüller's formula represents the molecular heat at constant volume as the product of a constant, $\lambda = 0.98835$, and ϑ, a quantity which he calls the "Wärmedimension," whose magnitude depends on the number of degrees of freedom possessed by the molecule. The "Wärmedimension" ϑ is defined by the formula $\vartheta = a + 2i$, where a represents the number of variable coördinates, at the most six in number, which determine the position of the molecule, and i represents the number of those coördinates, determining the position of the atoms in the molecule, which are variable and which determine, therefore, the number of degrees of freedom possessed by the molecule because of the peculiarities of its atomic structure.

By means of this formula I have calculated the values of i for the different substances from their observed molecular heats, and have then assigned such degrees of freedom to the atoms in their molecules as will reproduce these values. The interest in the results lies in the general simplicity of the schemes adopted, and in the analogies which appear between isomers and between compounds of similar character. In view of the complexity of these molecules, it seems to me that any kinetic explanation of their molecular heats can only be got by some such method.

8. *Degrees of Freedom of Solids.* — In the following table are given the schemes adopted for the solid molecules. Column I. contains their chemical formulas; Column III., the values of i calculated from Staigmüller's formula; Column II., the values of i obtained by assigning to each of the atoms whose symbol stands at the top of the lettered columns the number placed under it. The molecular heats of the isomers differ by 6. I have, therefore, supposed that one of the differences between isomers consists in this, that the molecule of the one is rigidly bound in the crystal while that of the other is free. The values of i in the case of the substances marked with a star (*) are obtained by setting a of the formula equal to zero.

Except in the cases of the liquid glycerine and liquid phenol

Substance.	I.	C.	H.	O.	N.	II.	III.
Dextrose.	$C_6H_{12}O_6$	1	1	1		24	25.5
Levulose.*	$C_6H_{12}O_6$	1	1	1		24	25
Mannite.	$C_6H_{14}O_6$	1	1	1		26	26
Dulcite.*	$C_6H_{14}O_6$	1	1	1		26	26
Resorcin.*	$C_6H_6O_2$	1	1	1		14	14.5
Hydroquinone.*	$C_6H_6O_2$	1	1	1		14	14
Pyrocatechin.	$C_6H_6O_2$	1	1	1		14	14.5
Urea.	CON_2H_4	1	1	1	1	8	7
Glycerine, liquid.	$C_3H_8O_3$	1	2	2		25	24
Phenol, liquid.	C_6H_6O	1	3	1		25	24

the supposition that the atoms have each one degree of freedom reproduces the numbers obtained from Staigmüller's formula very exactly.

9. *The Sugars.* — The sugars have been omitted from the above list. Their degrees of freedom cannot be reproduced on quite so simple a scheme. Some, but not all, of the atoms of one sort must have more than one degree of freedom. An examination of the structural formulas for the sugars shows that the carbon atoms appear in them in two groups, one of eight atoms and the other of four, differently connected with the other constituents of the molecule. We may assume that the atoms thus differently connected have different degrees of freedom. If this is done the schemes for the sugars are as follows:

Substance.	I.	C_4.	C_8.	H.	O.	II.	III.
Cane Sugar.*	$C_{12}H_{22}O_{11}$	1	2	1	1	53	52
Anhydrous Milk Sugar.*	$C_{12}H_{22}O_{11}$	2	1	1	1	49	50

The effect of the water of crystallization in ordinary milk sugar and in maltose cannot be represented without further hypotheses.

10. *Dextrose and Levulose.* — If we examine the structural formulas for dextrose and levulose, we find that one carbon atom in each is distinguished from the other five by being differently connected with the other constituents of the molecule. If we apply the same hypothesis as that just made for the sugars, by giving to this atom the degree of freedom 2, we find that i for dextrose and levulose becomes 25, and the most serious discrepancy of the table is removed.

11. *Degrees of Freedom in Solution.* — Turning now to the case of the same substances in solution, we may find i by the same formula. In this case a is always to be set equal to 6, as the molecules in solution must be considered free. The larger values of the molecular heats obtained in solution make it necessary to assign to some of the atoms a larger number of degrees of freedom than 1. The following table, with the same conventions as in §8, contains the schemes adopted.

Substance.	I.	C.	H.	O.	N.	II.	III.
Dextrose.	$C_6H_{12}O_6$	1	2	1		36	37
Levulose.	$C_6H_{12}O_6$	1	2	2		42	42
Mannite.	$C_6H_{14}O_6$	1	2	3		52	51.5
Dulcite.	$C_6H_{14}O_6$	1	2	2		46	46
Resorcin, water.	$C_6H_6O_2$	1	3	2		28	29
Hydroquinone, water.	$C_6H_6O_2$	1	3	2		28	29
Pyrocatechin, water.	$C_6H_6O_2$	2	3	2		34	35
" alcohol.	$C_6H_6O_2$	1	3	1		26	26
Phenol, water.	C_6H_6O	2	3	2		32	33
" alcohol.	C_6H_6O	1	3	1		25	23
Urea, water.	CON_2H_4	1	1	1	1	8	8
" alcohol.	CON_2H_4	1	2	1	1	12	11
Acetamide.	C_2H_5NO	1	2	2	1	15	15
Glycerine.	$C_3H_8O_3$	1	2	2		25	24

For the sugars we make the same supposition as before and obtain the following schemes.

Substance.	I.	C_4.	C8.	H.	O.	II.	III.
Cane Sugar.	$C_{12}H_{22}O_{11}$	1	2	2	1	75	74
Milk Sugar.	$C_{12}H_{22}O_{11}$	2	1	2	1	71	71
Maltose.	$H_{12}O_{22}C_{11}$	2	1	2	1	71	69

12. *Comments.* — I may direct attention to the following points which will be observed on an examination of the foregoing tables:

(*a*) The number of degrees of freedom which are assigned to the atoms never exceeds three, that is, the atoms of these sorts and in these relations can be conceived of as material points.

(*b*) The differences between the molecular heats of isomers may be ascribed to differences in the freedom of their atoms of one kind.

(*c*) The differences between the molecular heats of a substance

in water and in alcohol may be ascribed to differences in the degrees of freedom introduced by solution.

(*d*) The act of solution may be considered as involving a loosening of the bonds confining the atoms in the molecule, whereby the number of its degrees of freedom is increased.

The particular choice made, in the schemes here presented, of the numbers to be assigned to the several atoms, is not always the only one that may be made. It commends itself to me, however, because of the probability that the carbon atoms, around which the others are clustered, would be most restricted in their freedom, and so should have assigned to them, when possible, the lowest number.

13. *Degrees of Freedom of Liquids.* — A similar set of coincidences is exhibited by the molecular heats of organic liquids. A number of these are given in the following table, for which the data were taken from Landolt and Börnstein's tables. The molecular heats employed in the calculation of i by Staigmüller's formula are those given for the temperature nearest 0°, except when otherwise noted.

Substance.	I.	C.	H.	O.	II.	III.
Formic acid.	CH_2O_2	1	2	2	9	9
Acetic acid.	$C_2H_4O_2$	1	2	1	12	12
Methyl alcohol.	CH_4O	1	1	1	6	6.5
Ethyl alcohol.	C_2H_6O	1	1	1	9	9.5
" 160°		1	3	2	22	22.5
Propyl alcohol.	C_3H_8O	1	1	1	12	12.5
Isobutyl alcohol.	$C_4H_{10}O$	1	1	1	15	15
Isoamyl alcohol.	$C_5H_{12}O$	1	1	1	18	19
Ethyl ether.	$C_4H_{10}O$	1	1	1	15	15.5
" 120°		1	2	2	26	26.5
" 180°		1	3	2	36	35.5
Oil of Turpentine.	$C_{10}H_{16}$	1	1		26	25
Hexan.	C_6H_{14}	1	1		20	18.5
Heptan.	C_7H_{16}	1	1		23	21.5
Octan.	C_8H_{18}	1	1		26	26
Hexadecan.	$C_{16}H_{34}$	1	1		50	53

14. *Degrees of Freedom of Liquids.*—In the following table are given similar schemes for the degrees of freedom obtained from Schlamp's observations [1] on liquids at 100°. Some other values are

[1] Schlamp, Beiblätter, XX., 1896, p. 857.

inserted for other temperatures, taken from Landolt and Börnstein's tables.

Substance.	I.	C.	H.	O.	N.	II.	III.
Nitrobenzol, 8°	$C_6H_5O_2N$	1	2	1	1	19	18.5
" 100°		1	2	2	2	22	22
Aniline, 25°	C_6H_7N	1	2		1	21	21
" 100°		1	2		2	22	22.5
Benzol, 6°	C_6H_6	1	1			12	10
" 100°		1	2			18	16
Naphthaline.	$C_{10}H_8$	1	2			26	24.5
a-Naphthylamide.	$C_{10}H_9N$	1	2		3	31	31
Nitronaphthaline.	$C_{10}H_7NO_2$	1	2	2	3	31	31
p-Toluidine.	C_7H_9N	1	2		1	26	26
o-Toluidine.	C_7H_9N	1	2		1	26	26

15. *Comments.*—It appears on an examination of these tables for liquids that their degrees of freedom can be represented by schemes similar to those used before. In particular:

(*a*) The molecular heats of these liquids at low temperatures may be described in most cases by assigning one degree of freedom to each atom of the molecule.

(*b*) The change in the molecular heats as the temperature rises may be ascribed to an increase in the degrees of freedom of their atoms.

16. *Restatement of Hypothesis.*—The relations exhibited in all these tables may be otherwise expressed by saying that the molecular heat of an organic substance is the sum of the heat taken up by the molecule as a whole and of the atomic heats of the atoms, these atomic heats, which are different in different circumstances and in different combinations, being equal to 2λ multiplied by 1, 2, or 3, or within one per cent. to 2, 4, or 6. The heat taken up by the molecule as a whole is in most cases equal to 6λ, or within one per cent. to 6. The slight discrepancies which appear when this scheme is worked out may be explained by the distinction introduced by Jeans[1] between principal and subordinate degrees of freedom.

17. *Degrees of Freedom of Vapors.*—It is apparently a general rule that the molecular heats of vapors are less than those of the liquids from which they are formed. In terms of our present hy-

[1] Phil. Trans., Vol. 196, A, p. 397.

pothesis, the degrees of freedom of a molecule are more numerous in the liquid than in the vapor. This may be rendered possible by the comparative balancing of the forces acting on an atom in a liquid due to the presence of neighboring molecules. When the molecule escapes from the liquid on evaporation this action no longer takes place and the atoms fall back into more restricted relations. Calculations from such scanty data as I have at hand show that similar schemes to those already presented may be constructed for the vapors. The number of degrees of freedom, calculated from Staigmüller's formula, comes out so low in each case, that it is necessary to suppose that the carbon atoms in the molecules of these vapors lose their freedom entirely.

18. *Heats of Solution. Method.* — I have determined the heats of solution in water of the organic solids previously studied, to see if any relation could be shown to exist between them and the estimated change in their degrees of freedom brought about by solution. The determination was made with the silver cups of my calorimeter, fitted with covers for holding the thermometers and the silver paddles. A portion of the cover was hinged and could be raised to introduce the solid. Each cup was filled with 750 grams of distilled water at about the room temperature. The substance to be examined was finely powdered, except in a few cases of very soluble substances, 20 grams of it were weighed out, and a thermometer bulb covered with it. The cups were then placed in position and the temperatures of the water in both observed, until the change from minute to minute seemed uniform and the same in both. The temperature of the powder was then read, and the powder poured into one of the cups. The temperatures were then read at frequent intervals until by the uniform and equal rise of temperature in both cups it was evident that solution was complete. By the help of the control cup, in which the pure water changed its temperature steadily by reason of radiation and the agitation by the paddles, the change of temperature due to the same causes in the other cup could be determined, and a simple comparison with one of the temperatures observed in that cup, after solution was complete, gave the change of temperature due to the introduction of the solid substance. Knowing the specific heats of

the substance, both as solid and in solution, from my previous measurements, and knowing the heat capacity of the cup, it is easy to calculate the heat developed by the act of solution. In every case but that of glycerine heat was absorbed, or the heat of solution is negative.

19. *Heats of Solution. Results.* — The results of the experiments are given in the following table. Because of the small amount of material on hand, most of the substances were tested only once. From the results obtained when the experiments were repeated, it apppears that the results in general are not in error by more than one or two per cent.

In the table, Column I. contains the heat developed by 1 gram of

Substance.	I.	II.	III.
Cane Sugar.	− 3.6	− 1231	19.7
Hydrous milk sugar.	− 11.5	− 4140	21.6
Anhydrous milk sugar.	− 7.4	− 2530	19.2
Erdmann's milk sugar.	− 3.6	− 1231	19.3
Maltose.	− 9.75	− 3510	19.8
Dextrose.	− 13.02	− 2344	18.7
"	− 13.09	− 2356	18.9
Levulose.	− 10.6	− 1908	18.7
Mannite.	− 28.2	− 5132	18.7
Dulcite.	− 36.7	− 6679	18.9
Resorcin.	− 34.1	− 3751	18.2
"	− 35.3	− 3883	20.0
Hydroquinone.	− 41.2	− 4532	18.5
"	− 41.8	− 4598	20.0
Pyrocatechin.	− 31.3	− 3443	19.5
"	− 31.0	− 3410	19.0
Phenol.	− 30.6	− 2876	19.9
Urea.	− 60.4	− 3624	18.6
Glycerine.	+ 14.0	+ 1290	20.0

the substance dissolved in 37.5 grams of water; Column II., the product of this quantity of heat and the molecular weight of the substance, or the heat developed by a gram molecule of the substance dissolved in 37.5 times its weight of water; Column III., the temperature of the water into which the substance was introduced, given to the nearest tenth of a degree.

20. *Comments.* — It will be noticed, on an examination of these results, that the isomers behave differently with respect to the

heat developed by solution, just as they do with respect to their molecular heat. Even the isomers resorcin and hydroquinone, which in their specific heat relations are so similar, have different heats of solution. Except in the case of the isomers dextrose and levulose, the one of a set of isomers which dissolves with most difficulty has the largest negative heat of solution.

No general relation between the heats of solution and the estimated change in the number of degrees of freedom has been detected. The heats of solution, depending as they do on the work done in separating the molecules of the solid, and perhaps on the negative work done by combination of these molecules with those of the solvent, as well as on the change in the degrees of freedom within the molecule, should not of necessity exhibit any such relation.

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ON A MECHANICAL METHOD FOR TRACING THE CURVES $x^k y = \text{CONSTANT}$.

By Edgar Buckingham.

1. For students who are studying the properties of gases — particularly in connection with steam, gas, and air engines — it would be of advantage to have some means, less laborious than computation and plotting, for constructing the curves $x^k y = C$, which have, approximately, the shapes of the adiabatic lines in the (p, v) plane. Since I do not remember seeing any mention of an instrument for this purpose, I have devised one; and though I have not had facilities for constructing a working model, I think that the scheme may interest some readers of the Physical Review.

The first object in such a design is simplicity of construction. It is therefore obvious that the relative motions of the various parts of the instrument should, so far as possible, be motions of rotation, and that the number of sliding motions should be reduced to a minimum. For it is easier to do good work quickly on the turning lathe than on the planing, shaping, or milling machine. There must, furthermore, be the possibility of taking up, by springs, all lost motion or back-lash, at the various contacts.

2. By differentiating the equation $x^k y = C$, we get

$$kx^{k-1}ydx + x^k dy = 0,$$

whence

$$\frac{dy}{dx} = -k\frac{y}{x}. \tag{1}$$

The curve must therefore be drawn in such a way that its slope is equal to $-k(y/x)$.

To this end, the tracing point is to be mounted in a carriage rolling over the tracing surface on guide-wheels; and these guide-wheels are to be so directed, by rotation of the carriage about an

axis through the tracing point and perpendicular to the plane *oxy*, that the motion of advance of the tracing point is the proper one, satisfying equation (1). The problem is, then, to move some bar about a fixed axis so that it shall have the direction required for the tangent to the curve at the point occupied by the tracer; and then, to find a means of transmitting this *direction* to the tracing carriage, so that the planes of rotation of the guide-wheels shall be always parallel to this bar. The problem may be solved in the following manner:

A straight bar AB is pivoted at the origin of coördinates O. A rail RR', perpendicular to the axis of y and below the origin, acts as guide to a carriage S, which carries a double pivot with its axis perpendicular to the plane *oxy* and passing through the line RR'. The lower part of this double pivot turns directly in a bearing in the carriage, and has, perpendicular to its axis, a hole through which the bar AB can slide freely but without shaking. The upper part of the pivot turns in a bearing in the lower part and has, perpendicular to the common axis of both parts of the pivot, a hole through which the straight bar PC slides while rotating about the pivot P. The pivot P is fixed for any given value of the constant k, but can be set, by motion along the negative axis of y, so as to give the distance $\overline{RP}$ any value between $\overline{RO}$ and 1.67 $\overline{RO}$.

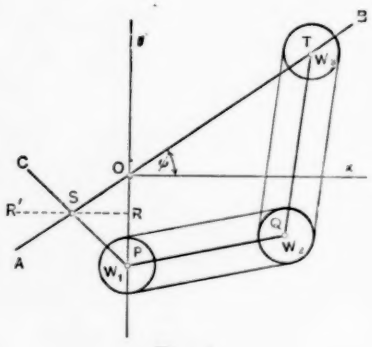

Fig. 1.

Upon the bar AB is a slide carrying on the side nearest the tracing surface, a pivot T. The tracing point, supported and guided in its motion by two coaxial guide-wheels, touches the tracing surface in the intersection with that surface, of the axis of the pivot T. The frame which contains the bearings of the guide-wheels turns upon the pivot T, concentrically with the tracing point.

An arm PQ, pivoted at P, is jointed at Q to the arm QT which turns about T; thus the pivot T does a double duty but has not the complications of that at S. Concentric with P is a grooved pulley

W_1, rigidly connected with the arm PC and sharing its rotations. Concentric with Q is a doubly grooved pulley W_2, rotating freely on a prolongation of the pivot Q. Concentric with T, and rigidly connected with the carriage which holds the tracing point and guide-wheels, is a third grooved pulley W_3. These three pulleys are of equal diameter. An inextensible band passes, with friction, around W_1 and W_2, and another similar band around W_2 and W_3.

3. Suppose P to have been so fixed on $-oy$, that $RP/RO = k$. If the bar AB rotates about O, the carriage S slides along the rail RR', while AB itself slides through S, as does PC. In this way, the intersection of the projections of PC and AB on the plane oxy lies constantly on the fixed projection of the line RR'. For the sake of brevity, we will now regard the figure as a purely geometrical one drawn in the plane oxy.

Since AB passes through the tracing point T, we have

$$\text{tan angle } \overline{RSO} = \frac{RO}{SR} = \frac{y}{x}.$$

But

$$\text{tan angle } \overline{RSP} = \frac{RP}{SR} = -k\frac{RO}{SR} = -k\frac{y}{x}.$$

Therefore by equation (1) we have

$$\text{tan angle } \overline{RSP} = \frac{dy}{dx};$$

or, in other words, CSP has the direction of the tangent drawn to the required curve at the point occupied by the tracer T.

If the frame which holds the guide-wheels and tracing point be so rotated about the pivot T, that the axis of the guide-wheel is at every instant perpendicular to PC, the tracing point will be so directed over the surface, that as the bar AB rotates about O, the required curve is described. This, as is easily seen, is accomplished by the bands about W_1, W_2, W_3, if there is no slip and if the planes of rotation of the guide-wheels have once been set parallel to PC. The whole object of the arms PQ and QT is to provide an extensible band-connection between P and T. If AB be now rotated about O, the tracing point, while sliding with its pivot along AB,

will trace the curve $x^k y = C$; and the value of C may be varied by lifting T from the tracing surface, sliding it along the radial arm AB, and finally readjusting the axis of the guide-wheels so that it shall again be perpendicular to PC.

4. Though this instrument is simple compared with several other preliminary schemes which occurred to me, it has several places where possible lost motion must be taken up. The arms PQ and QT may be made telescopic and extended by compressed spiral springs so as to keep the bands (fine brass wire) always in a state of tension; or, if the pulleys are small in diameter, the bands may be made elastic by the insertion of spiral springs. A spiral spring acting toward the right on the carriage S will take up the lost motion of AB and PC at S. A coiled spring about the pivot T, between the slide and the rotating tracing carriage, will take up lost motion of rotation. The looseness of S on RR', and of T on AB may be taken up in various ways too obvious to need mention.

The mechanical construction may be simplified by making the main radial arm AB, and the secondary or tangent arm PC, double, each consisting of a pair of parallel bars (brass tubes). In this case, only a single pivot is needed at S and it may be fixed rigidly upon the carriage which slides along RR'. Two slides, one on AB and the other on PC, each bored to fit over the pivot S will then accomplish the desired result.

It is obvious that whatever be the available lengths of the rail RR' and of the arms OA and PC, the action of the mechanism will become stiff and inaccurate when RS is large. Hence, supposing RR, OA and PC to be indefinitely long, the minimum value of φ to be attained in practice, will decrease as the accuracy of construction increases and as the friction — especially the sliding friction at S and T — diminishes.

A similar limit is set to the maximum practicable value of φ by the sliding friction at T; but, on the whole, the instrument as here planned will be more effective for large than for small values of φ. If the parts of the curve nearer the axis is of x, that is, those for which φ is small, are of the greater importance, the scheme may easily be so altered as to interchange the limitations to which φ is subject in the vicinity of O and $\pi/2$.

It may be noticed, in conclusion, that since the pivot P may be so placed as to make RP equal to (or even less than) RO, the instrument may be adjusted for drawing the equilateral hyperbolas $xy = C$, which give an approximate representation of the isothermal lines of the gases in the pv plane, for values of the temperature which are above and not too near the critical temperature.

PROCEEDINGS

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AMERICAN PHYSICAL SOCIETY.

C_p IN LIQUIDS AND THE a OF VAN DER WAALS IN THE CASE OF WATER.

BY EDWIN H. HALL.

THE specific heat, C_v, of liquids at constant volume is a subject which deserves more attention than it generally receives. For example, Nernst on p. 234 of his *Theoretische Chemie*, 2d edition, says: "If a fluid were kept at constant volume during heating, then (provided the theory of van der Waals is strictly applicable to it) its specific heat would have to be just as great as that of its vapor, the latter also, of course, being taken at constant volume," and he appears to argue in favor of the general truth of this relation, without, however, giving any specific instance of C_v, except in the case of water at 4° C., where, obviously, $C_v = C_p$.

It is easy to get a convenient formula for C_v in terms of C_p, e, the coefficient of expansion at constant pressure, k, the coefficient of compressibility at constant temperature, v, the specific volume, and T, the absolute temperature. This formula[1] is,

$$(1) \qquad C_v = C_p - \frac{e^2 v\, T}{k}, \text{ or } C_v = C_p + \frac{\left(\frac{dv}{dT}\right)_p^2}{\left(\frac{dv}{dp}\right)_T} T.$$

If we apply this formula to the case of water at temperatures above 4°C., we find that $C_v < C_p$. At 50° C., for example, $C_p \div C_v = 1.03 +$; but at this temperature, and probably at much higher temperatures, C_v for liquid water is still much greater than C_v for water vapor.

[1] Mr. L. D. Hill, of Cambridge, finds that Combes (*Theorie de La Chaleur*, 1863) worked out numerical values of C_v for various liquids, using a less simple and less convenient formula. The attention of the author has been called to a passage on p. 81 of Kirchhoff's *Lectures on Heat*, where the formula here written is given and applied to the case of water.

By imagining unit mass of a liquid put through a very small cycle consisting of an isothermal, an isopiestic, and an isometric, it is easy, with the use of equation (1), to derive the following expression for ΔE, the internal latent heat of expansion at constant temperature through a small increase of volume Δv:

$$\Delta E = \left(\frac{e}{k} T - p\right) \Delta v. \tag{2}$$

The argument thus far has been thermodynamic and rigid. It has now brought us to a place of uncertainty and of interesting speculation. If we assume that ΔE is merely the work of overcoming molecular attraction and that, as in the theory of van der Waals, this attraction is expressed by a/v^2, where a is a constant, we can find a value of a in the case of an expanding liquid by means of the equation,

$$\Delta E = \left(\frac{e}{k} T - p\right) \Delta v = a \left(\frac{1}{v} - \frac{1}{v + \Delta v}\right). \tag{3}$$

Applied to water at 15° C. this equation gives approximately $a = 933 \times 10^6$. If now, using this value of a, we calculate the amount of energy required to overcome the molecular attraction in the evaporation of water at 15° C., we find that only about one twenty-fifth part of λ, the internal latent heat of evaporation is thus accounted for. If we make a similar calculation at 50° C., we find, from the expansion of the liquid, $a = 3{,}720 \times 10^6$, large enough to account for nearly one sixth part of λ at this temperature. At 100° we get in the same way $a =$ perhaps $6{,}400 \times 10^6$, accounting for nearly one third of λ. At the critical temperature the discrepancy should, apparently, entirely disappear.

The theory of van der Waals requires that, for any given temperature, a and the mean energy of *translatory* molecular motion shall be constants. It apparently does not require that the *whole* energy of molecular motion shall be a function of temperature only. Apparently we could account for the thermodynamic relations of water and water vapor by means of the following assumptions:

1. That, at temperatures below the critical, a is independent of volume but increases with temperature.

2. That, at temperatures below the critical, the total energy of molecular motion increases during evaporation, though the energy of translatory molecular motion may remain constant during this change of state.

It is not suggested that this is the only way of accounting for the behavior of water.

THE HEAT OF VAPORIZATION OF NITROGEN.

BY J. S. SHEARER.

IN connection with the determination of the heat of vaporization of air it was found desirable to find this constant for each constituent. The result for oxygen has been reported at a previous meeting. For atmospheric nitrogen it was found that 49.8 calories per gram are absorbed in vaporization. As the result for oxygen was 60.9 it follows that the heat of vaporization of air should increase as it becomes richer in oxygen.

THE HEAT OF VAPORIZATION OF AIR AS RELATED TO ITS COMPOSITION.

BY J. S. SHEARER AND F. R. STRAYER.

SINCE there is a considerable difference in the heats of vaporization of oxygen and nitrogen the writers sought to determine whether or not the energy used is the same as would be required to vaporize the same amount of the unmixed liquids. The composition of the vapor phase was found by pyrogallol absorption, and percentages of oxygen and total vapor produced were plotted as coördinates. The area of this curve gives the total oxygen in the original liquid air, and the composition of the liquid phase at any point is readily found. The results would indicate that the heat required is probably the same as would be required to vaporize the constituents separately.

NOTE.

Carl Anton Bjerknes. — Carl Anton Bjerknes was born in Christiania on the 24th of October, 1825. He interested himself at an early day both in mathematics and physics, but at the University of Christiania, where he studied in 1844, there was no advanced instruction in these subjects. He was therefore compelled to devote himself to mineralogy instead and in 1849 he was appointed to a position in the Silver Mining Works at Kongsberg. During this period he continued his mathematical studies as best he could and during 1855 and 1857 he received a public stipend with which to study in Paris and Goettingen. Upon his return he became a docent in mathematics at the University of Christiania. In 1863 he was appointed reader and in 1866 professor of applied mathematics. In 1869 he was transferred to the chair of pure mathematics which he occupied up to the time of his death.

Bjerknes's older published work was in the realm of pure mathematics, but he is best known for his investigations in hydrodynamics. A study of the works of Euler had early led him into the ranks of the opposition to the law of *action at a distance*, which in his younger days was almost universally accepted. While a student at Goettingen he attended the lectures of Lejeune-Dirichlet, in the course of which the solution of the hydrodynamic problem of the movement of a sphere in a fluid was presented. This solution led to the conclusion, very surprising at first sight, that the sphere would move through the fluid with constant velocity and without meeting with resistance; a result which brought Bjerknes to the conception that the principle of inertia would hold with the same exactitude in a space filled with fluid as in vacuo. He was thus led to attack the problem of the simultaneous movement of several spheres in a fluid and to the question whether their movements would not mutually influence each other. Such an influence would obviously present the appearance of action at a distance between sphere and sphere and it was his purpose to determine whether these apparent actions at a distance were not analogous to those observed in nature. Greatly hindered by illness and other circumstances he was able to attain to the solution of this problem for the first time in 1868 when he found that such apparent actions at a distance really took place between the spheres and that with the single striking exception that the law of equality of action and reaction did not hold, they possessed all the fundamental properties which

we are accustomed to assume in the ordinary mechanics of action at a distance in nature. Further penetration into the nature of the forces which Bjerknes had discovered was first attained in 1875, at which time he found that these forces could be divided into two parts, one of which produced an invisible, the other visible motions. For the latter the principle of equal action and reaction held true so that this force possessed all the fundamental properties ascribed to the forces of nature in mechanics. This force which produced visible motions showed moreover the most extraordinary similarity to those producing action at a distance in electrostatics and magnetism; but always with opposite sign.

With the applications of this remarkable hydrodynamic analogy to electrical and magnetic phenomena Bjerknes subsequently occupied himself up to the time of his death; extending the comparison further and further and verifying all the principal results by experiment. The experimental work confirmed in the most striking manner the results of his computations and showed that these hydrodynamic actions at a distance have in every respect the appearance of interaction between electrified or magnetized bodies but always with the direction of the force reversed. Bjerknes was not, unfortunately, to have the satisfaction of laying his results before the scientific world except through the chance exhibition of some of his experiments or by means of preliminary communications concerning the same. The latest mathematical discussion which he published on this subject was printed in 1876. His extraordinary conscientiousness and excessive criticism of his own labors, taken in connection with the extraordinary difficulty of the subject, prevented him from working out in complete form the great treatise on his discoveries which he had planned and the editing of this work was accordingly taken up by his son, Professor V. Bjerknes,[1] of Stockholm. The investigations have not nearly all of them as yet appeared in the published form, but a few months before his death, which occurred on the 20th of March, 1903, Bjerknes had the pleasure of seeing the first portions of the complete work in print.

V. BJERKNES.

[1] V. Bjerknes: Vorlesungen über hydrodynamische Fernkräfte nach C. A. Bjerknes' Theorie; Leipzig, 1900–02.

NEW BOOKS.

Handbuch der Spectroscopie. Von H. KAYSER. Zweiter Band; pp. xi + 696. S. Hirzel, Leipzig, 1902.

The second volume of Professor Kayser's Manual fully meets the expectations aroused by the appearance of Volume I. We find the same breadth of treatment and the same extraordinary completeness in dealing with the extensive literature of this subject. No contribution of any importance seems to have escaped the notice of the author and what is more remarkable, when one considers the enormous number of papers which have appeared, each piece of work has been read and its value estimated with fairness and good judgment. The author gives his opinion with great frankness and freedom, but his criticisms are tempered throughout with fairness and his dictum concerning many difficult points which have been in the past or still are subjects of dispute will command respect, and in general, acquiescence on the part of those who read his admirable summaries. A good example of Professor Kayser's ability to deal with such matters may be seen in his discussion of the relative claims of Balfour Stewart and Kirchhoff in the establishment of the law of emission. Stewart's failure to produce a conclusive and completely generalized demonstration of what we know as Kirchhoff's law is clearly brought out but the value and the significance of his extended labors in this field are fully and appreciatively recognized. Another instance of the author's skill in the handling of a vexed question is found in the review of the literature dealing with the phenomena of the first appearance of visible radiation from a hot body (pages 41–48). Kayser points out quite rightly that the later observations of Weber, Stenger, Ebert, Emden, etc., do not lead to an abandonment of the earlier views but that they serve rather to establish certain physiological phenomena of vision unrecognized by Draper, Kirchhoff and their contemporaries. There can be little doubt, in view of the newer researches, of the correctness of Violle's contention that the reception of light, with gradual rise of intensities precedes the reception of color. Nor is it strange considering the enormously greater luminosity of the rays lying in the middle of the spectrum that these should first be capable of producing color sensation in spite of the greater energy of the longer wave-lengths. The comparison of the isochromatic curves of intensity shows, it is true, in the case of some radiating bodies that the growth in intensity with tem-

perature does not increase uniformly as the wave-length decreases, in strict accordance with the views of Draper but that in certain regions intensity increases faster than in others so that the phenomenon is a more complicated one than was once imagined.

In his brief discussion of the phenomena generally classed under the head of fluorescence the author makes use of the very appropriate terminology invented by Wiedemann and describes these types of radiation under the title of *luminescence*. In the important chapters on the principle of Doppler and upon the Zeeman effect he has availed himself of the services of Dr. Konen and Professor Runge respectively; a choice which justifies itself in the admirable treatment given to these subjects.

One of the most important chapters in this volume is that which deals with the structure of line spectra, the grouping of lines, their occurrence in systems or series and the laws which appear to govern the arrangement of such lines. This is a domain in which we are likely to see important developments in the near future and the gathering together of the work already done is at this time peculiarly serviceable and useful.

All students in light and especially those who have occasion to work with the spectrum will look forward to the remaining volumes of Professor Kayser's hand-book with the greatest interest and confidence. No such comprehensive work has ever been attempted in this field of research. Its value to those who are striving to advance our knowledge of spectroscopy is beyond estimate. E. L. N.

Electrical Problems. By W. L. HOOPER and R. T. WELLS. Pp. i + 170. Boston, Ginn & Company, 1902.

This work comprises a particularly clear set of problems well suited to give concrete illustration to all the more important principles of electricity and magnetism. The problems are well arranged and selected so as to give to abstract relations a reality which can otherwise be attained only by a long experience with electrical apparatus and engineering calculations. No theory is given, but suggestions are made which for the most part are exceeding apt. Practice with the problems in this book should give the student not only facility and accuracy in making calculations, but a much clearer conception of the fundamental principles of electrical engineering.

F. BEDELL.